

# Application of the Lasso on genomic data

## Part 3

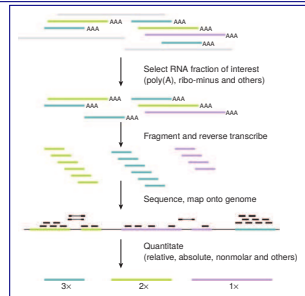
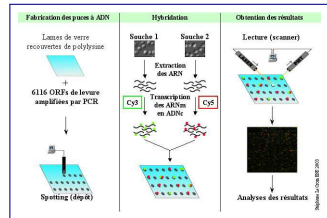
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ECAS, High Dimensional Statistics Course, October 2017

# Genomics and high dimensional statistics

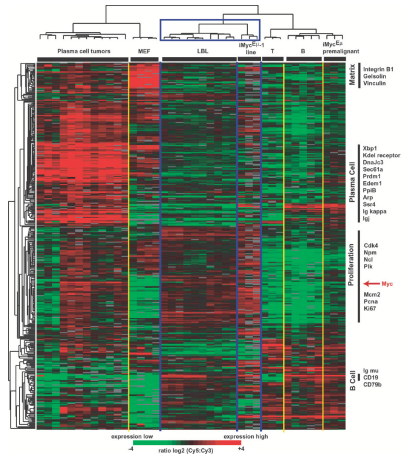
- 1990-2000: DNA microarrays
- 2000-: massive parallel sequencing: RNA-Seq, chIP-Seq
- NGS : next generation sequencing



Genome wide molecular portraits of cells

# The high throughput point of view

- Before high throughput technologies, gene expression was quantified gene by gene
- High throughput technologies completely changed the perspectives of biologists
- In one experiment, one has access of the measurements of all transcripts in a cell (tissue)



## Structure of datasets

	statut	exon <sub>1</sub>	exon <sub>2</sub>	...	exon <sub>p</sub>	Age	Sexe	Glycemia
$i = 1$	0	10000	50		0	38	F	0.8
$i = 2$	1	10000	30		1	15	M	0.2
$\vdots$								
$i = N$	1	20000	25		3	90	F	1.5

For each individual

- Status (discrete/continuous)
- gene expression measurements (counts ou continuous)
- Clinical data

### Goal

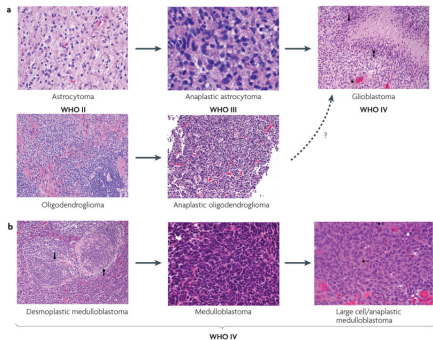
Explain the variations of a given response with gene expression measurements.

## Example of statistical tasks

- Experimental Design
- Differential Analysis (multiple testing)
- Unsupervised Classification (clustering)
- Phenotype prediction (supervised)

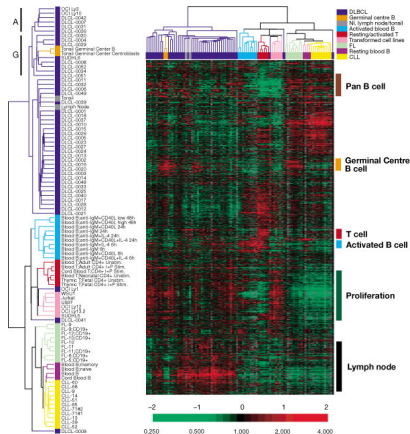
Standard statistical tasks but need to be revisited to account for high dimension

# New classifications and personalized medicine



Nature Reviews | Cancer

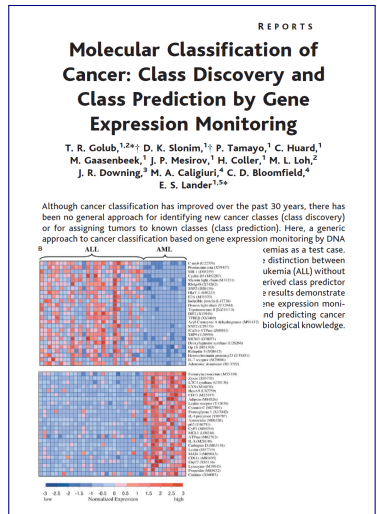
Classification based on histological data



Classification on molecular data

# Towards new predictions ?

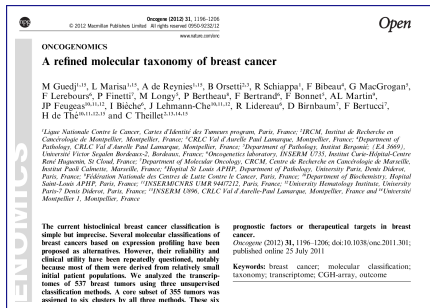
- In 1999 an article proposes to predict the molecular status of leukemia patients using genomic signatures
- The number of individuals is 38 for 6817 genes !
- Methodological developments for genomic signatures and prediction



# Data set: Breast Cancer Relapse

- level expression of 54613 genes for 294 patients affected by breast cancer.
- Y: relapse after 5 years (binary)
- 214 patients without relapse and 80 with a relapse.
- Preselection of 5000 genes
- What is the (ada)Lasso estimation of this data, estimation ? Prediction

► Lasso on genomic data





## (Sparse) PCA to visualize the data

- $\mathbf{X}_{[n \times p]}$  measurements of  $p$  genes over  $n$  individuals (centered)
- Find a  $K$  dimensional subspace to represent the data
- Define  $\mathbf{U}_{n \times K}$  the coordinates of the individuals in the new space
- Define  $\mathbf{V}_{p \times K}$  the coordinates of the variables in the new space (loadings)

Approximate  $\mathbf{X} \simeq \mathbf{UV}'$  by linear projection

# Principal components

- $\mathbf{t}_k$  is a linear combination of the observed variables

$$\mathbf{t}_k = \mathbf{X}\mathbf{w}_k$$

- $\mathbf{w}_k \in \mathbb{R}^p$ : contributions of the variables to the component
- The objective function is the empirical variance of the components  $\hat{\mathbf{V}}(\mathbf{t}_k)$  (under orthogonality constraints)

$$\hat{\mathbf{w}}_k = \arg \max_{\mathbf{w}_k \in \mathbb{R}^p} \{(\mathbf{w}_k \mathbf{X})' \mathbf{X} \mathbf{w}_k\}$$

- The solution is explicit and is given by the  $K$  eigen vectors of the empirical variance of  $\mathbf{X}$  (centered)

# Sparse PCA

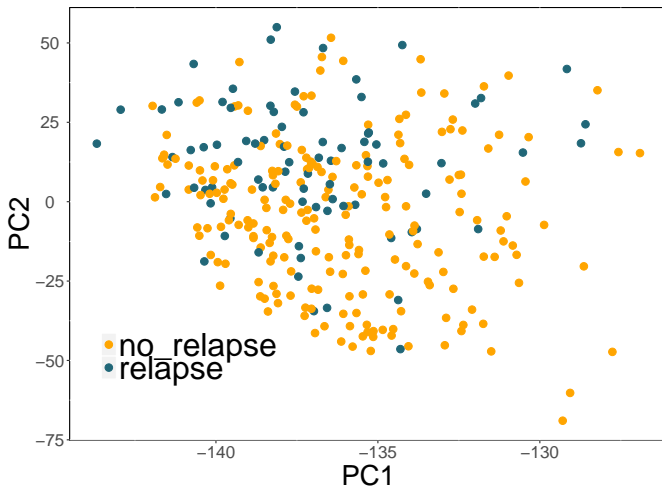
► Sparse PCA coding

- Some variables may contribute poorly to components ([Ex of Sparse PCA paper](#))
- $\mathbf{w}_k$  are assumed to be sparse

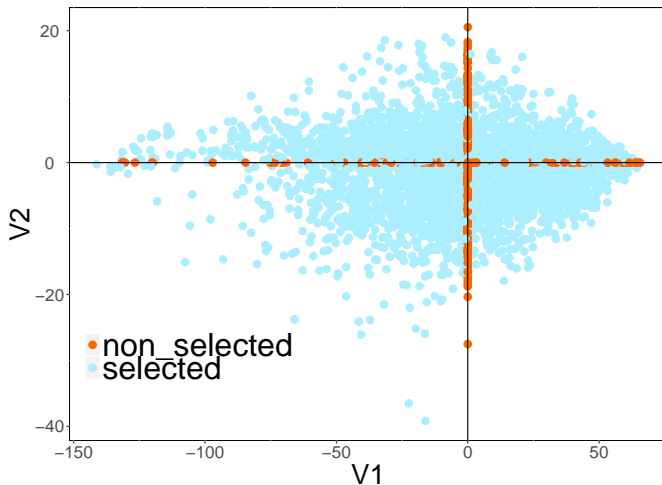
$$\hat{\mathbf{w}}_k(\lambda) = \arg \max_{\mathbf{w}_k \in \mathbb{R}^p} \{ (\mathbf{w}_k \mathbf{X})' \mathbf{X} \mathbf{w}_k - \lambda \sum_k \|\mathbf{w}_k\|_1 \}$$

- Also known as sparse coding or [sparse matrix factorization](#)
- Need to calibrate the hyperparameter by cross validation

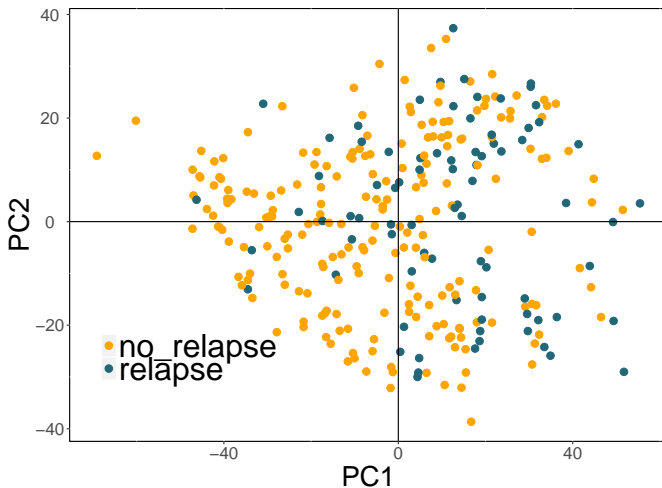
## (Sparse) PCA to visualize the individuals



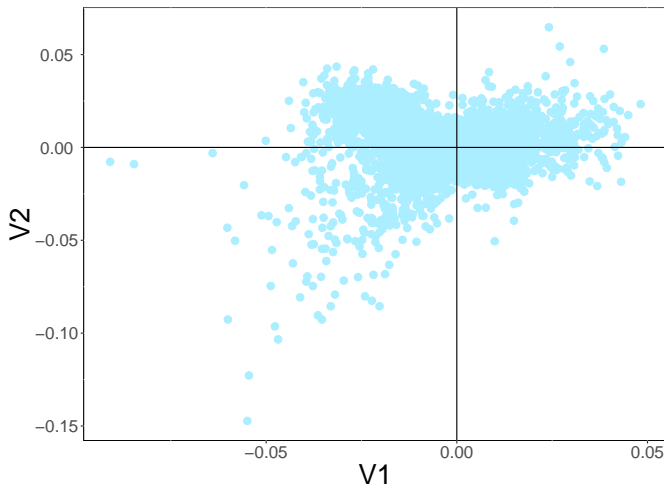
## (Sparse) PCA to visualize the variables



## (Sparse) PCA to visualize the individuals



## (Sparse) PCA to visualize the variables



# (Sparse) PLS for supervised dimension reduction

## ► Sparse PLS coding

- What if there is a response vector  $\mathbf{Y}$  ?
- Instead of reduction dimension based on  $\widehat{\mathbf{V}}(\mathbf{t}_k)$ , use  $\widehat{\mathbf{Cov}}(\mathbf{t}_k, \mathbf{Y})$

$$\widehat{\mathbf{w}}_k(\lambda) = \arg \max_{\mathbf{w}_k \in \mathbb{R}^p \mid \|\mathbf{w}_k\|_2^2 = 1} \{(\mathbf{w}_k \mathbf{X})' \mathbf{Y} - \lambda \sum_k \|\mathbf{w}_k\|_1\}$$

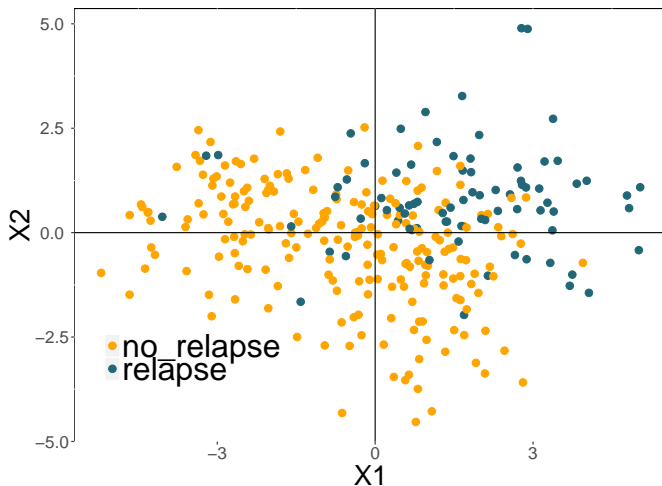
- Can be reformulated as a regression problem:  $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{E}$
- Considering  $\mathbf{T} = \mathbf{X}\mathbf{W}$  the matrix of principal components, PLS performs a regression of  $\mathbf{Y}$  on  $\mathbf{T}$

$$\mathbf{Y} = \mathbf{T}\boldsymbol{\gamma} + \widetilde{\mathbf{E}}, \text{ with } \widehat{\boldsymbol{\beta}} = \mathbf{W}\widehat{\boldsymbol{\gamma}}$$

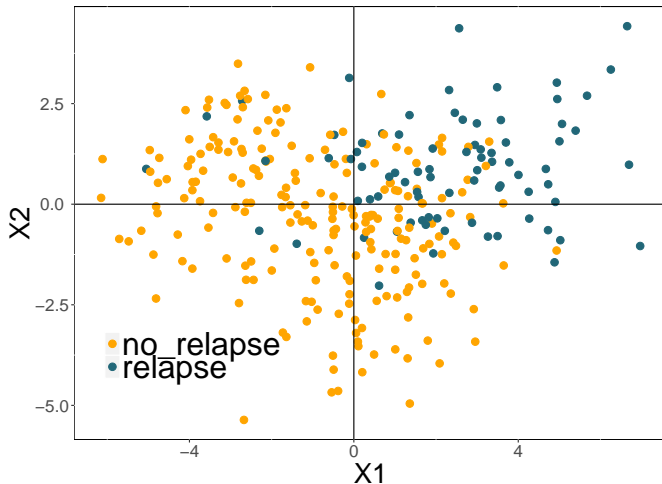
- Can be adapted to logistic regression ( $\mathbf{Y}$  binary, tricky computations, [\*sparse logistic PLS paper\*](#)).



## (Sparse) PLS to visualize the individuals



## (Sparse) PLS to visualize the individuals



## (Sparse) PLS to visualize the variables

