

# 1 gamma.gee: Generalized Estimating Equation for Gamma Regression

The GEE gamma is similar to standard gamma regression (appropriate when you have an uncensored, positive-valued, continuous dependent variable such as the time until a parliamentary cabinet falls). Unlike in gamma regression, GEE gamma allows for dependence within clusters, such as in longitudinal data, although its use is not limited to just panel data. GEE models make no distributional assumptions but require three specifications: a mean function, a variance function, and a “working” correlation matrix for the clusters, which models the dependence of each observation with other observations in the same cluster. The “working” correlation matrix is a  $T \times T$  matrix of correlations, where  $T$  is the size of the largest cluster and the elements of the matrix are correlations between within-cluster observations. The appeal of GEE models is that it gives consistent estimates of the parameters and consistent estimates of the standard errors can be obtained using a robust “sandwich” estimator even if the “working” correlation matrix is incorrectly specified. If the “working” correlation matrix is correctly specified, GEE models will give more efficient estimates of the parameters. GEE models measure population-averaged effects as opposed to cluster-specific effects (See (author?) [4]).

## 1.0.1 Syntax

```
> z.out <- zelig(Y ~ X1 + X2, model = "gamma.gee",
               id = "X3", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

where `id` is a variable which identifies the clusters. The data should be sorted by `id` and should be ordered within each cluster when appropriate.

## 1.0.2 Additional Inputs

- **robust**: defaults to TRUE. If TRUE, consistent standard errors are estimated using a “sandwich” estimator.

Use the following arguments to specify the structure of the “working” correlations within clusters:

- **corstr**: defaults to "independence". It can take on the following arguments:
  - Independence (**corstr** = "independence"):  $\text{cor}(y_{it}, y_{it'}) = 0, \forall t, t'$  with  $t \neq t'$ . It assumes that there is no correlation within the clusters and the model becomes equivalent to standard gamma regression. The “working” correlation matrix is the identity matrix.

- Fixed (**corstr** = "fixed"): If selected, the user must define the “working” correlation matrix with the **R** argument rather than estimating it from the model.
- Stationary  $m$  dependent (**corstr** = "stat\_M\_dep"):

$$\text{cor}(y_{it}, y_{it'}) = \begin{cases} \alpha_{|t-t'|} & \text{if } |t-t'| \leq m \\ 0 & \text{if } |t-t'| > m \end{cases}$$

If (**corstr** = "stat\_M\_dep"), you must also specify  $Mv = m$ , where  $m$  is the number of periods  $t$  of dependence. Choose this option when the correlations are assumed to be the same for observations of the same  $|t - t'|$  periods apart for  $|t - t'| \leq m$ .

Sample “working” correlation for Stationary 2 dependence ( $Mv=2$ )

$$\begin{pmatrix} 1 & \alpha_1 & \alpha_2 & 0 & 0 \\ \alpha_1 & 1 & \alpha_1 & \alpha_2 & 0 \\ \alpha_2 & \alpha_1 & 1 & \alpha_1 & \alpha_2 \\ 0 & \alpha_2 & \alpha_1 & 1 & \alpha_1 \\ 0 & 0 & \alpha_2 & \alpha_1 & 1 \end{pmatrix}$$

- Non-stationary  $m$  dependent (**corstr** = "non\_stat\_M\_dep"):

$$\text{cor}(y_{it}, y_{it'}) = \begin{cases} \alpha_{tt'} & \text{if } |t-t'| \leq m \\ 0 & \text{if } |t-t'| > m \end{cases}$$

If (**corstr** = "non\_stat\_M\_dep"), you must also specify  $Mv = m$ , where  $m$  is the number of periods  $t$  of dependence. This option relaxes the assumption that the correlations are the same for all observations of the same  $|t - t'|$  periods apart.

Sample “working” correlation for Non-stationary 2 dependence ( $Mv=2$ )

$$\begin{pmatrix} 1 & \alpha_{12} & \alpha_{13} & 0 & 0 \\ \alpha_{12} & 1 & \alpha_{23} & \alpha_{24} & 0 \\ \alpha_{13} & \alpha_{23} & 1 & \alpha_{34} & \alpha_{35} \\ 0 & \alpha_{24} & \alpha_{34} & 1 & \alpha_{45} \\ 0 & 0 & \alpha_{35} & \alpha_{45} & 1 \end{pmatrix}$$

- Exchangeable (**corstr** = "exchangeable"):  $\text{cor}(y_{it}, y_{it'}) = \alpha, \forall t, t'$  with  $t \neq t'$ . Choose this option if the correlations are assumed to be the same for all observations within the cluster.

Sample “working” correlation for Exchangeable

$$\begin{pmatrix} 1 & \alpha & \alpha & \alpha & \alpha \\ \alpha & 1 & \alpha & \alpha & \alpha \\ \alpha & \alpha & 1 & \alpha & \alpha \\ \alpha & \alpha & \alpha & 1 & \alpha \\ \alpha & \alpha & \alpha & \alpha & 1 \end{pmatrix}$$

- Stationary  $m$ th order autoregressive (`corstr = "AR-M"`): If (`corstr = "AR-M"`), you must also specify `Mv = m`, where  $m$  is the number of periods  $t$  of dependence. For example, the first order autoregressive model (AR-1) implies  $\text{cor}(y_{it}, y_{it'}) = \alpha^{|t-t'|}$ ,  $\forall t, t'$  with  $t \neq t'$ . In AR-1, observation 1 and observation 2 have a correlation of  $\alpha$ . Observation 2 and observation 3 also have a correlation of  $\alpha$ . Observation 1 and observation 3 have a correlation of  $\alpha^2$ , which is a function of how 1 and 2 are correlated ( $\alpha$ ) multiplied by how 2 and 3 are correlated ( $\alpha$ ). Observation 1 and 4 have a correlation that is a function of the correlation between 1 and 2, 2 and 3, and 3 and 4, and so forth.

Sample “working” correlation for Stationary AR-1 (`Mv=1`)

$$\begin{pmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ \alpha & 1 & \alpha & \alpha^2 & \alpha^3 \\ \alpha^2 & \alpha & 1 & \alpha & \alpha^2 \\ \alpha^3 & \alpha^2 & \alpha & 1 & \alpha \\ \alpha^4 & \alpha^3 & \alpha^2 & \alpha & 1 \end{pmatrix}$$

- Unstructured (`corstr = "unstructured"`):  $\text{cor}(y_{it}, y_{it'}) = \alpha_{tt'}$ ,  $\forall t, t'$  with  $t \neq t'$ . No constraints are placed on the correlations, which are then estimated from the data.
- `Mv`: defaults to 1. It specifies the number of periods of correlation and only needs to be specified when `corstr` is `"stat_M_dep"`, `"non_stat_M_dep"`, or `"AR-M"`.
- `R`: defaults to `NULL`. It specifies a user-defined correlation matrix rather than estimating it from the data. The argument is used only when `corstr` is `"fixed"`. The input is a  $T \times T$  matrix of correlations, where  $T$  is the size of the largest cluster.

### 1.0.3 Examples

#### 1. Example with Exchangeable Dependence

Attaching the sample turnout dataset:

```
> data(coalition)
```

Sorted variable identifying clusters

```
> coalition$cluster <- c(rep(c(1:62),5),rep(c(63),4))
> sorted.coalition <- coalition[order(coalition$cluster),]
```

Estimating model and presenting summary:

```
> z.out <- zelig(duration ~ fract + numst2, model = "gamma.gee", id = "cluster", data =
```

```

      (Intercept)      fract      numst2
-0.0129597411  0.0001148931 -0.0173874664

```

How to cite this model in Zelig:

Patrick Lam. 2013.

"gamma.gee: General Estimating Equation for Gamma Regression"

in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software,"

<http://gking.harvard.edu/zelig>

```
> summary(z.out)
```

```

GEE:  GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)

```

Model:

```

Link:                      Reciprocal
Variance to Mean Relation: Gamma
Correlation Structure:     Exchangeable

```

Call:

```

gee(formula = formula, id = id, corstr = corstr, family = Gamma,
    data = data, .hook = robust.gee.hook)

```

Summary of Residuals:

```

      Min      1Q      Median      3Q      Max
-51.662849 -10.922635  -3.338295   9.384375  33.481595

```

Coefficients:

```

              Estimate   Naive S.E.   Naive z   Robust S.E.   Robust z
(Intercept) -0.0129634095 1.328203e-02 -0.9760113 0.0126829873 -1.022110
fract        0.0001149138 1.719796e-05  6.6818299 0.0000147418  7.795104
numst2       -0.0174009000 5.886821e-03 -2.9559076 0.0062943755 -2.764516

```

Estimated Scale Parameter: 0.6291527

Number of Iterations: 1

Working Correlation

```

      [,1]      [,2]      [,3]      [,4]      [,5]
[1,] 1.000000000 -0.008042939 -0.008042939 -0.008042939 -0.008042939
[2,] -0.008042939 1.000000000 -0.008042939 -0.008042939 -0.008042939
[3,] -0.008042939 -0.008042939 1.000000000 -0.008042939 -0.008042939
[4,] -0.008042939 -0.008042939 -0.008042939 1.000000000 -0.008042939
[5,] -0.008042939 -0.008042939 -0.008042939 -0.008042939 1.000000000

```

Setting the explanatory variables at their default values (mode for factor

variables and mean for non-factor variables), with numst2 set to the vector  
0 = no crisis, 1 = crisis.

```
> x.low <- setx(z.out, numst2 = 0)
> x.high <- setx(z.out, numst2 = 1)
```

Simulate quantities of interest

```
> s.out <- sim(z.out, x = x.low, x1 = x.high)
> summary(s.out)
```

Model: gamma.gee

Number of simulations: 1000

Values of X

```
(Intercept)      fract numst2
1           1 718.8121        0
attr("assign")
[1] 0 1 2
```

Values of X1

```
(Intercept)      fract numst2
1           1 718.8121        1
attr("assign")
[1] 0 1 2
```

Expected Values (for x): E(Y|X)

mean	sd	50%	2.5%	97.5%
14.468	1.114	14.399	12.552	16.916

Expected Values (for x1): E(Y|X1)

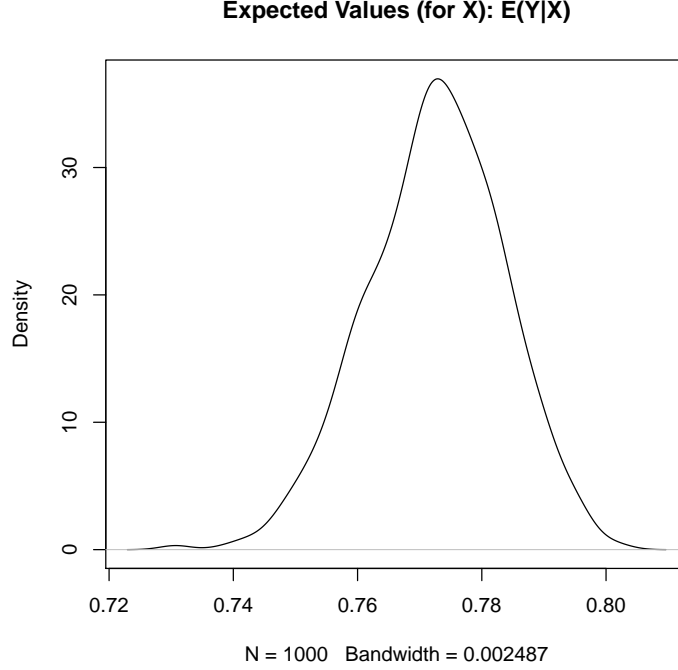
mean	sd	50%	2.5%	97.5%
19.207	1.135	19.169	17.207	21.602

First Differences: E(Y|X1) - E(Y|X)

mean	sd	50%	2.5%	97.5%
4.74	1.617	4.726	1.41	7.882

Generate a plot of quantities of interest:

```
> plot(s.out)
```



#### 1.0.4 The Model

Suppose we have a panel dataset, with  $Y_{it}$  denoting the positive-valued, continuous dependent variable for unit  $i$  at time  $t$ .  $Y_i$  is a vector or cluster of correlated data where  $y_{it}$  is correlated with  $y_{it'}$  for some or all  $t, t'$ . Note that the model assumes correlations within  $i$  but independence across  $i$ .

- The *stochastic component* is given by the joint and marginal distributions

$$\begin{aligned} Y_i &\sim f(y_i \mid \lambda_i) \\ Y_{it} &\sim g(y_{it} \mid \lambda_{it}) \end{aligned}$$

where  $f$  and  $g$  are unspecified distributions with means  $\lambda_i$  and  $\lambda_{it}$ . GEE models make no distributional assumptions and only require three specifications: a mean function, a variance function, and a correlation structure.

- The *systematic component* is the *mean function*, given by:

$$\lambda_{it} = \frac{1}{x_{it}\beta}$$

where  $x_{it}$  is the vector of  $k$  explanatory variables for unit  $i$  at time  $t$  and  $\beta$  is the vector of coefficients.

- The *variance function* is given by:

$$V_{it} = \lambda_{it}^2 = \frac{1}{(x_{it}\beta)^2}$$

- The *correlation structure* is defined by a  $T \times T$  “working” correlation matrix, where  $T$  is the size of the largest cluster. Users must specify the structure of the “working” correlation matrix *a priori*. The “working” correlation matrix then enters the variance term for each  $i$ , given by:

$$V_i = \phi A_i^{\frac{1}{2}} R_i(\alpha) A_i^{\frac{1}{2}}$$

where  $A_i$  is a  $T \times T$  diagonal matrix with the variance function  $V_{it} = \lambda_{it}^2$  as the  $t$ th diagonal element,  $R_i(\alpha)$  is the “working” correlation matrix, and  $\phi$  is a scale parameter. The parameters are then estimated via a quasi-likelihood approach.

- In GEE models, if the mean is correctly specified, but the variance and correlation structure are incorrectly specified, then GEE models provide consistent estimates of the parameters and thus the mean function as well, while consistent estimates of the standard errors can be obtained via a robust “sandwich” estimator. Similarly, if the mean and variance are correctly specified but the correlation structure is incorrectly specified, the parameters can be estimated consistently and the standard errors can be estimated consistently with the sandwich estimator. If all three are specified correctly, then the estimates of the parameters are more efficient.
- The robust “sandwich” estimator gives consistent estimates of the standard errors when the correlations are specified incorrectly only if the number of units  $i$  is relatively large and the number of repeated periods  $t$  is relatively small. Otherwise, one should use the “naïve” model-based standard errors, which assume that the specified correlations are close approximations to the true underlying correlations. See [?] for more details.

### 1.0.5 Quantities of Interest

- All quantities of interest are for marginal means rather than joint means.
- The method of bootstrapping generally should not be used in GEE models. If you must bootstrap, bootstrapping should be done within clusters, which is not currently supported in Zelig. For conditional prediction models, data should be matched within clusters.
- The expected values (qi\$ev) for the GEE gamma model is the mean:

$$E(Y) = \lambda_c = \frac{1}{x_c\beta},$$

given draws of  $\beta$  from its sampling distribution, where  $x_c$  is a vector of values, one for each independent variable, chosen by the user.

- The first difference (`qi$fd`) for the GEE gamma model is defined as

$$FD = \Pr(Y = 1 \mid x_1) - \Pr(Y = 1 \mid x).$$

- In conditional prediction models, the average expected treatment effect (`att.ev`) for the treatment group is

$$\frac{1}{\sum_{i=1}^n \sum_{t=1}^T tr_{it}} \sum_{i:tr_{it}=1}^n \sum_{t:tr_{it}=1}^T \{Y_{it}(tr_{it} = 1) - E[Y_{it}(tr_{it} = 0)]\},$$

where  $tr_{it}$  is a binary explanatory variable defining the treatment ( $tr_{it} = 1$ ) and control ( $tr_{it} = 0$ ) groups. Variation in the simulations are due to uncertainty in simulating  $E[Y_{it}(tr_{it} = 0)]$ , the counterfactual expected value of  $Y_{it}$  for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to  $tr_{it} = 0$ .

### 1.0.6 Output Values

The output of each `Zelig` command contains useful information which you may view. For example, if you run `z.out <- zelig(y ~ x, model = "gamma.gee", id, data)`, then you may examine the available information in `z.out` by using `names(z.out)`, see the `coefficients` by using `z.out$coefficients`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output object `z.out`, you may extract:
  - `coefficients`: parameter estimates for the explanatory variables.
  - `residuals`: the working residuals in the final iteration of the fit.
  - `fitted.values`: the vector of fitted values for the systemic component.
  - `linear.predictors`: the vector of  $x_{it}\beta$
  - `max.id`: the size of the largest cluster.
- From `summary(z.out)`, you may extract:
  - `coefficients`: the parameter estimates with their associated standard errors,  $p$ -values, and  $z$ -statistics.
  - `working.correlation`: the “working” correlation matrix
- From the `sim()` output object `s.out`, you may extract quantities of interest arranged as matrices indexed by simulation  $\times$   $x$ -observation (for more than one  $x$ -observation). Available quantities are:
  - `qi$ev`: the simulated expected values for the specified values of  $x$ .



- `qi$fd`: the simulated first difference in the expected probabilities for the values specified in `x` and `x1`.
- `qi$att.ev`: the simulated average expected treatment effect for the treated from conditional prediction models.

## How To Cite the *gamma.gee* Zelig model

Patrick Lam. 2007. “gamma.gee: Generalized Estimating Equation for Gamma Regression,” in Kosuke Imai, Gary King, and Olivia Lau, “Zelig: Everyone’s Statistical Software,” <http://gking.harvard.edu/zelig>.

## How to Cite the Zelig Software Package

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Imai, Kosuke, Gary King, and Olivia Lau. (2008). “Toward A Common Framework for Statistical Analysis and Development.” *Journal of Computational and Graphical Statistics*, Vol. 17, No. 4 (December), pp. 892-913.

## See also

The `gee` function is part of the `gee` package by Vincent J. Carey, ported to R by Thomas Lumley and Brian Ripley. Advanced users may wish to refer to `help(gee)` and `help(family)`. Sample data are from [1].

## 2 `logit.gee`: Generalized Estimating Equation for Logistic Regression

The GEE logit estimates the same model as the standard logistic regression (appropriate when you have a dichotomous dependent variable and a set of explanatory variables). Unlike in logistic regression, GEE logit allows for dependence within clusters, such as in longitudinal data, although its use is not limited to just panel data. The user must first specify a “working” correlation matrix for the clusters, which models the dependence of each observation with other observations in the same cluster. The “working” correlation matrix is a  $T \times T$  matrix of correlations, where  $T$  is the size of the largest cluster and the elements of the matrix are correlations between within-cluster observations. The appeal of GEE models is that it gives consistent estimates of the parameters and consistent estimates of the standard errors can be obtained using a robust “sandwich” estimator even if the “working” correlation matrix is

incorrectly specified. If the “working” correlation matrix is correctly specified, GEE models will give more efficient estimates of the parameters. GEE models measure population-averaged effects as opposed to cluster-specific effects (See (author?) [4]).

### 2.0.7 Syntax

```
> z.out <- zelig(Y ~ X1 + X2, model = "logit.gee",
               id = "X3", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

where `id` is a variable which identifies the clusters. The data should be sorted by `id` and should be ordered within each cluster when appropriate.

### 2.0.8 Additional Inputs

- **robust**: defaults to `TRUE`. If `TRUE`, consistent standard errors are estimated using a “sandwich” estimator.

Use the following arguments to specify the structure of the “working” correlations within clusters:

- **corstr**: defaults to `"independence"`. It can take on the following arguments:

- Independence (**corstr** = `"independence"`):  $\text{cor}(y_{it}, y_{it'}) = 0, \forall t, t'$  with  $t \neq t'$ . It assumes that there is no correlation within the clusters and the model becomes equivalent to standard logistic regression. The “working” correlation matrix is the identity matrix.
- Fixed (**corstr** = `"fixed"`): If selected, the user must define the “working” correlation matrix with the `R` argument rather than estimating it from the model.
- Stationary  $m$  dependent (**corstr** = `"stat_M_dep"`):

$$\text{cor}(y_{it}, y_{it'}) = \begin{cases} \alpha_{|t-t'|} & \text{if } |t - t'| \leq m \\ 0 & \text{if } |t - t'| > m \end{cases}$$

If (**corstr** = `"stat_M_dep"`), you must also specify `Mv = m`, where  $m$  is the number of periods  $t$  of dependence. Choose this option when the correlations are assumed to be the same for observations of the same  $|t - t'|$  periods apart for  $|t - t'| \leq m$ .

Sample “working” correlation for Stationary 2 dependence (`Mv=2`)

$$\begin{pmatrix} 1 & \alpha_1 & \alpha_2 & 0 & 0 \\ \alpha_1 & 1 & \alpha_1 & \alpha_2 & 0 \\ \alpha_2 & \alpha_1 & 1 & \alpha_1 & \alpha_2 \\ 0 & \alpha_2 & \alpha_1 & 1 & \alpha_1 \\ 0 & 0 & \alpha_2 & \alpha_1 & 1 \end{pmatrix}$$

- Non-stationary  $m$  dependent (`corstr` = "non\_stat\_M\_dep"):

$$\text{cor}(y_{it}, y_{it'}) = \begin{cases} \alpha_{tt'} & \text{if } |t - t'| \leq m \\ 0 & \text{if } |t - t'| > m \end{cases}$$

If (`corstr` = "non\_stat\_M\_dep"), you must also specify  $Mv = m$ , where  $m$  is the number of periods  $t$  of dependence. This option relaxes the assumption that the correlations are the same for all observations of the same  $|t - t'|$  periods apart.

Sample “working” correlation for Non-stationary 2 dependence  
( $Mv=2$ )

$$\begin{pmatrix} 1 & \alpha_{12} & \alpha_{13} & 0 & 0 \\ \alpha_{12} & 1 & \alpha_{23} & \alpha_{24} & 0 \\ \alpha_{13} & \alpha_{23} & 1 & \alpha_{34} & \alpha_{35} \\ 0 & \alpha_{24} & \alpha_{34} & 1 & \alpha_{45} \\ 0 & 0 & \alpha_{35} & \alpha_{45} & 1 \end{pmatrix}$$

- Exchangeable (`corstr` = "exchangeable"):  $\text{cor}(y_{it}, y_{it'}) = \alpha, \forall t, t'$  with  $t \neq t'$ . Choose this option if the correlations are assumed to be the same for all observations within the cluster.

Sample “working” correlation for Exchangeable

$$\begin{pmatrix} 1 & \alpha & \alpha & \alpha & \alpha \\ \alpha & 1 & \alpha & \alpha & \alpha \\ \alpha & \alpha & 1 & \alpha & \alpha \\ \alpha & \alpha & \alpha & 1 & \alpha \\ \alpha & \alpha & \alpha & \alpha & 1 \end{pmatrix}$$

- Stationary  $m$ th order autoregressive (`corstr` = "AR-M"): If (`corstr` = "AR-M"), you must also specify  $Mv = m$ , where  $m$  is the number of periods  $t$  of dependence. For example, the first order autoregressive model (AR-1) implies  $\text{cor}(y_{it}, y_{it'}) = \alpha^{|t-t'|}, \forall t, t'$  with  $t \neq t'$ . In AR-1, observation 1 and observation 2 have a correlation of  $\alpha$ . Observation 2 and observation 3 also have a correlation of  $\alpha$ . Observation 1 and observation 3 have a correlation of  $\alpha^2$ , which is a function of how 1 and 2 are correlated ( $\alpha$ ) multiplied by how 2 and 3 are correlated ( $\alpha$ ). Observation 1 and 4 have a correlation that is a function of the correlation between 1 and 2, 2 and 3, and 3 and 4, and so forth.

Sample “working” correlation for Stationary AR-1 ( $Mv=1$ )

$$\begin{pmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ \alpha & 1 & \alpha & \alpha^2 & \alpha^3 \\ \alpha^2 & \alpha & 1 & \alpha & \alpha^2 \\ \alpha^3 & \alpha^2 & \alpha & 1 & \alpha \\ \alpha^4 & \alpha^3 & \alpha^2 & \alpha & 1 \end{pmatrix}$$

- Unstructured (`corstr = "unstructured"`):  $\text{cor}(y_{it}, y_{it'}) = \alpha_{tt'}, \forall t, t'$  with  $t \neq t'$ . No constraints are placed on the correlations, which are then estimated from the data.
- `Mv`: defaults to 1. It specifies the number of periods of correlation and only needs to be specified when `corstr` is `"stat_M_dep"`, `"non_stat_M_dep"`, or `"AR-M"`.
- `R`: defaults to `NULL`. It specifies a user-defined correlation matrix rather than estimating it from the data. The argument is used only when `corstr` is `"fixed"`. The input is a  $T \times T$  matrix of correlations, where  $T$  is the size of the largest cluster.

## 2.0.9 Examples

### 1. Example with Stationary 3 Dependence

Attaching the sample turnout dataset:

```
> data(turnout)
```

Variable identifying clusters

```
> turnout$cluster <- rep(c(1:200), 10)
```

Sorting by cluster

```
> sorted.turnout <- turnout[order(turnout$cluster),]
```

Estimating parameter values for the logistic regression:

```
> z.out1 <- zelig(vote ~ race + educate, model = "logit.gee", id = "cluster", data = sorted.turnout)
```

(Intercept)	racewhite	educate
-1.2189037	0.5022257	0.1610007

How to cite this model in Zelig:

Patrick Lam. 2013.

"logit.gee: General Estimating Equation for Logistic Regression"

in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software,"

<http://gking.harvard.edu/zelig>

Setting values for the explanatory variables to their default values:

```
> x.out1 <- setx(z.out1)
```

Simulating quantities of interest:

```
> s.out1 <- sim(z.out1, x = x.out1)
```

```
> summary(s.out1)
```

```

Model:  logit.gee
Number of simulations:  1000

Values of X
  (Intercept) racewhite  educate
1           1           1 12.06675
attr(,"assign")
[1] 0 1 2
attr(,"contrasts")
attr(,"contrasts")$race
[1] "contr.treatment"

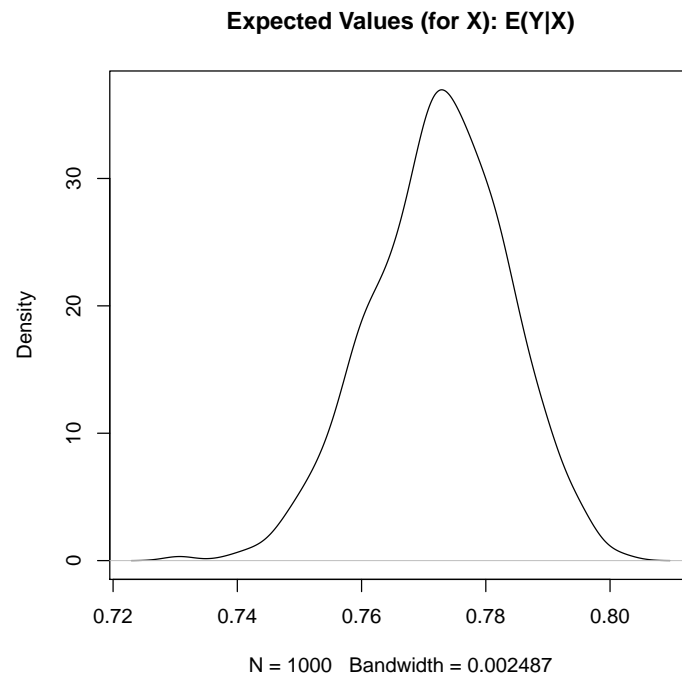
```

```

Expected Values (for x): E(Y|X)
  mean    sd  50%  2.5% 97.5%
0.773 0.011 0.773 0.751 0.795

```

```
> plot(s.out1)
```



## 2. Simulating First Differences

Estimating the risk difference (and risk ratio) between low education (25th

percentile) and high education (75th percentile) while all the other variables held at their default values.

```
> x.high <- setx(z.out1, educate = quantile(turnout$educate, prob = 0.75))
> x.low <- setx(z.out1, educate = quantile(turnout$educate, prob = 0.25))

> s.out2 <- sim(z.out1, x = x.high, x1 = x.low)

> summary(s.out2)
```

Model: logit.gee

Number of simulations: 1000

Values of X

```
(Intercept) racewhite educate
1          1          1      14
attr("assign")
[1] 0 1 2
attr("contrasts")
attr("contrasts")$race
[1] "contr.treatment"
```

Values of X1

```
(Intercept) racewhite educate
1          1          1      10
attr("assign")
[1] 0 1 2
attr("contrasts")
attr("contrasts")$race
[1] "contr.treatment"
```

Expected Values (for x):  $E(Y|X)$

mean	sd	50%	2.5%	97.5%
0.823	0.011	0.823	0.8	0.845

Expected Values (for x1):  $E(Y|X1)$

mean	sd	50%	2.5%	97.5%
0.71	0.015	0.71	0.679	0.739

First Differences:  $E(Y|X1) - E(Y|X)$

mean	sd	50%	2.5%	97.5%
-0.113	0.012	-0.113	-0.136	-0.091

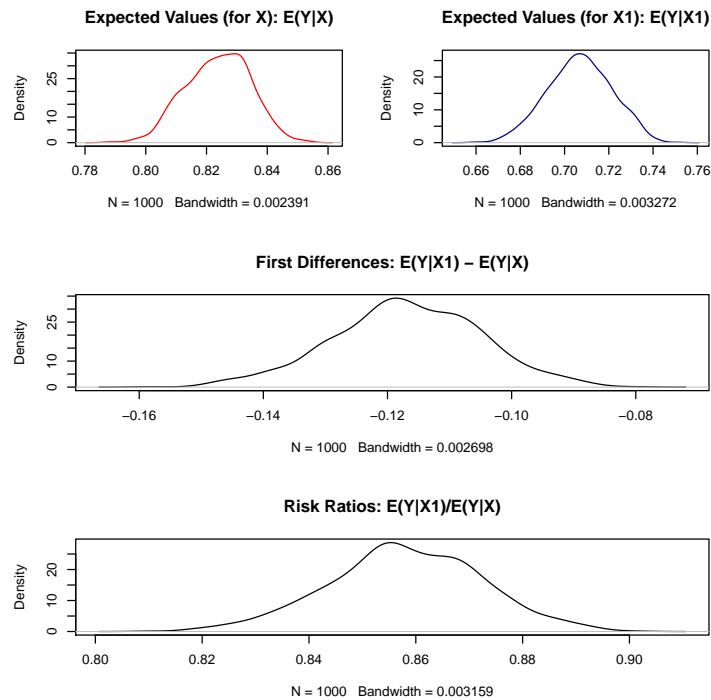
Risk Ratios:  $E(Y|X1)/E(Y|X)$

```

mean    sd   50%  2.5% 97.5%
0.862 0.014 0.862 0.834 0.889

```

```
> plot(s.out2)
```



### 3. Example with Fixed Correlation Structure

User-defined correlation structure

```

> corr.mat <- matrix(rep(0.5,100), nrow=10, ncol=10)
> diag(corr.mat) <- 1

```

Generating empirical estimates:

```

> z.out2 <- zelig(vote ~ race + educate, model = "logit.gee", id = "cluster", data = so

(Intercept)  racewhite    educate
-1.2189037   0.5022257    0.1610007

```

How to cite this model in Zelig:

Patrick Lam. 2013.

"logit.gee: General Estimating Equation for Logistic Regression"

in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software,"  
<http://gking.harvard.edu/zelig>

Viewing the regression output:

```
> summary(z.out2)
```

```
GEE:  GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)
```

```
Model:
Link:                      Logit
Variance to Mean Relation: Binomial
Correlation Structure:     Fixed
```

```
Call:
gee(formula = formula, id = id, corstr = corstr, family = binomial(link = "logit"),
    data = data, R = R, .hook = robust.gee.hook)
```

```
Summary of Residuals:
      Min       1Q   Median       3Q      Max
-0.9067826 -0.3018991  0.2112738  0.2390951  0.7887027
```

```
Coefficients:
              Estimate Naive S.E.   Naive z Robust S.E.  Robust z
(Intercept) -1.3171233 0.16878117 -7.803733  0.22423028 -5.873976
racewhite    0.5593612 0.10125663  5.524193  0.14572484  3.838475
educate      0.1596174 0.01220733 13.075544  0.01657297  9.631189
```

```
Estimated Scale Parameter: 0.9574347
Number of Iterations: 3
```

```
Working Correlation
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
[1,]  1.0  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5
[2,]  0.5  1.0  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5
[3,]  0.5  0.5  1.0  0.5  0.5  0.5  0.5  0.5  0.5  0.5
[4,]  0.5  0.5  0.5  1.0  0.5  0.5  0.5  0.5  0.5  0.5
[5,]  0.5  0.5  0.5  0.5  1.0  0.5  0.5  0.5  0.5  0.5
[6,]  0.5  0.5  0.5  0.5  0.5  1.0  0.5  0.5  0.5  0.5
[7,]  0.5  0.5  0.5  0.5  0.5  0.5  1.0  0.5  0.5  0.5
[8,]  0.5  0.5  0.5  0.5  0.5  0.5  0.5  1.0  0.5  0.5
[9,]  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  1.0  0.5
[10,] 0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  1.0
```

## 2.0.10 The Model

Suppose we have a panel dataset, with  $Y_{it}$  denoting the binary dependent variable for unit  $i$  at time  $t$ .  $Y_i$  is a vector or cluster of correlated data where



$y_{it}$  is correlated with  $y_{it'}$  for some or all  $t, t'$ . Note that the model assumes correlations within  $i$  but independence across  $i$ .

- The *stochastic component* is given by the joint and marginal distributions

$$\begin{aligned} Y_i &\sim f(y_i | \pi_i) \\ Y_{it} &\sim g(y_{it} | \pi_{it}) \end{aligned}$$

where  $f$  and  $g$  are unspecified distributions with means  $\pi_i$  and  $\pi_{it}$ . GEE models make no distributional assumptions and only require three specifications: a mean function, a variance function, and a correlation structure.

- The *systematic component* is the *mean function*, given by:

$$\pi_{it} = \frac{1}{1 + \exp(-x_{it}\beta)}$$

where  $x_{it}$  is the vector of  $k$  explanatory variables for unit  $i$  at time  $t$  and  $\beta$  is the vector of coefficients.

- The *variance function* is given by:

$$V_{it} = \pi_{it}(1 - \pi_{it})$$

- The *correlation structure* is defined by a  $T \times T$  “working” correlation matrix, where  $T$  is the size of the largest cluster. Users must specify the structure of the “working” correlation matrix *a priori*. The “working” correlation matrix then enters the variance term for each  $i$ , given by:

$$V_i = \phi A_i^{\frac{1}{2}} R_i(\alpha) A_i^{\frac{1}{2}}$$

where  $A_i$  is a  $T \times T$  diagonal matrix with the variance function  $V_{it} = \pi_{it}(1 - \pi_{it})$  as the  $t$ th diagonal element,  $R_i(\alpha)$  is the “working” correlation matrix, and  $\phi$  is a scale parameter. The parameters are then estimated via a quasi-likelihood approach.

- In GEE models, if the mean is correctly specified, but the variance and correlation structure are incorrectly specified, then GEE models provide consistent estimates of the parameters and thus the mean function as well, while consistent estimates of the standard errors can be obtained via a robust “sandwich” estimator. Similarly, if the mean and variance are correctly specified but the correlation structure is incorrectly specified, the parameters can be estimated consistently and the standard errors can be estimated consistently with the sandwich estimator. If all three are specified correctly, then the estimates of the parameters are more efficient.
- The robust “sandwich” estimator gives consistent estimates of the standard errors when the correlations are specified incorrectly only if the number of units  $i$  is relatively large and the number of repeated periods  $t$  is relatively small. Otherwise, one should use the “naïve” model-based standard errors, which assume that the specified correlations are close approximations to the true underlying correlations. See [?] for more details.

### 2.0.11 Quantities of Interest

- All quantities of interest are for marginal means rather than joint means.
- The method of bootstrapping generally should not be used in GEE models. If you must bootstrap, bootstrapping should be done within clusters, which is not currently supported in Zelig. For conditional prediction models, data should be matched within clusters.
- The expected values (`qi$ev`) for the GEE logit model are simulations of the predicted probability of a success:

$$E(Y) = \pi_c = \frac{1}{1 + \exp(-x_c\beta)},$$

given draws of  $\beta$  from its sampling distribution, where  $x_c$  is a vector of values, one for each independent variable, chosen by the user.

- The first difference (`qi$fd`) for the GEE logit model is defined as

$$FD = \Pr(Y = 1 \mid x_1) - \Pr(Y = 1 \mid x).$$

- The risk ratio (`qi$rr`) is defined as

$$RR = \Pr(Y = 1 \mid x_1) / \Pr(Y = 1 \mid x).$$

- In conditional prediction models, the average expected treatment effect (`att.ev`) for the treatment group is

$$\frac{1}{\sum_{i=1}^n \sum_{t=1}^T tr_{it}} \sum_{i:tr_{it}=1}^n \sum_{t:tr_{it}=1}^T \{Y_{it}(tr_{it} = 1) - E[Y_{it}(tr_{it} = 0)]\},$$

where  $tr_{it}$  is a binary explanatory variable defining the treatment ( $tr_{it} = 1$ ) and control ( $tr_{it} = 0$ ) groups. Variation in the simulations are due to uncertainty in simulating  $E[Y_{it}(tr_{it} = 0)]$ , the counterfactual expected value of  $Y_{it}$  for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to  $tr_{it} = 0$ .

### 2.0.12 Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run `z.out <- zelig(y ~ x, model = "logit.gee", id, data)`, then you may examine the available information in `z.out` by using `names(z.out)`, see the coefficients by using `z.out$coefficients`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output object `z.out`, you may extract:

- `coefficients`: parameter estimates for the explanatory variables.
  - `residuals`: the working residuals in the final iteration of the fit.
  - `fitted.values`: the vector of fitted values for the systemic component,  $\pi_{it}$ .
  - `linear.predictors`: the vector of  $x_{it}\beta$
  - `max.id`: the size of the largest cluster.
- From `summary(z.out)`, you may extract:
    - `coefficients`: the parameter estimates with their associated standard errors,  $p$ -values, and  $z$ -statistics.
    - `working.correlation`: the “working” correlation matrix
  - From the `sim()` output object `s.out`, you may extract quantities of interest arranged as matrices indexed by simulation  $\times$  `x`-observation (for more than one `x`-observation). Available quantities are:
    - `qi$ev`: the simulated expected probabilities for the specified values of `x`.
    - `qi$fd`: the simulated first difference in the expected probabilities for the values specified in `x` and `x1`.
    - `qi$rr`: the simulated risk ratio for the expected probabilities simulated from `x` and `x1`.
    - `qi$att.ev`: the simulated average expected treatment effect for the treated from conditional prediction models.

## How To Cite the *logit.gee* Zelig Model

Patrick Lam. 2007. “logit.gee: Generalized Estimating Equation for Logit Regression,” in Kosuke Imai, Gary King, and Olivia Lau, “Zelig: Everyone’s Statistical Software,” <http://gking.harvard.edu/zelig>.

## How to Cite the Zelig Software Package

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Imai, Kosuke, Gary King, and Olivia Lau. (2008). “Toward A Common Framework for Statistical Analysis and Development.” *Journal of Computational and Graphical Statistics*, Vol. 17, No. 4 (December), pp. 892-913.

## See also

The `gee` function is part of the `gee` package by Vincent J. Carey, ported to R by Thomas Lumley and Brian Ripley. Advanced users may wish to refer to `help(gee)` and `help(family)`. Sample data are from [2].

## 3 `normal.gee`: Generalized Estimating Equation for Normal Regression

The GEE normal estimates the same model as the standard normal regression. Unlike in normal regression, GEE normal allows for dependence within clusters, such as in longitudinal data, although its use is not limited to just panel data. The user must first specify a “working” correlation matrix for the clusters, which models the dependence of each observation with other observations in the same cluster. The “working” correlation matrix is a  $T \times T$  matrix of correlations, where  $T$  is the size of the largest cluster and the elements of the matrix are correlations between within-cluster observations. The appeal of GEE models is that it gives consistent estimates of the parameters and consistent estimates of the standard errors can be obtained using a robust “sandwich” estimator even if the “working” correlation matrix is incorrectly specified. If the “working” correlation matrix is correctly specified, GEE models will give more efficient estimates of the parameters. GEE models measure population-averaged effects as opposed to cluster-specific effects (See (author?) [4]).

### 3.0.13 Syntax

```
> z.out <- zelig(Y ~ X1 + X2, model = "normal.gee",
               id = "X3", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

where `id` is a variable which identifies the clusters. The data should be sorted by `id` and should be ordered within each cluster when appropriate.

### 3.0.14 Additional Inputs

- **robust**: defaults to `TRUE`. If `TRUE`, consistent standard errors are estimated using a “sandwich” estimator.

Use the following arguments to specify the structure of the “working” correlations within clusters:

- **corstr**: defaults to `"independence"`. It can take on the following arguments:
  - Independence (`corstr = "independence"`):  $\text{cor}(y_{it}, y_{it'}) = 0, \forall t, t'$  with  $t \neq t'$ . It assumes that there is no correlation within the clusters

and the model becomes equivalent to standard normal regression. The “working” correlation matrix is the identity matrix.

- Fixed (`corstr = "fixed"`): If selected, the user must define the “working” correlation matrix with the `R` argument rather than estimating it from the model.
- Stationary  $m$  dependent (`corstr = "stat_M_dep"`):

$$\text{cor}(y_{it}, y_{it'}) = \begin{cases} \alpha_{|t-t'|} & \text{if } |t-t'| \leq m \\ 0 & \text{if } |t-t'| > m \end{cases}$$

If (`corstr = "stat_M_dep"`), you must also specify  $Mv = m$ , where  $m$  is the number of periods  $t$  of dependence. Choose this option when the correlations are assumed to be the same for observations of the same  $|t - t'|$  periods apart for  $|t - t'| \leq m$ .

Sample “working” correlation for Stationary 2 dependence ( $Mv=2$ )

$$\begin{pmatrix} 1 & \alpha_1 & \alpha_2 & 0 & 0 \\ \alpha_1 & 1 & \alpha_1 & \alpha_2 & 0 \\ \alpha_2 & \alpha_1 & 1 & \alpha_1 & \alpha_2 \\ 0 & \alpha_2 & \alpha_1 & 1 & \alpha_1 \\ 0 & 0 & \alpha_2 & \alpha_1 & 1 \end{pmatrix}$$

- Non-stationary  $m$  dependent (`corstr = "non_stat_M_dep"`):

$$\text{cor}(y_{it}, y_{it'}) = \begin{cases} \alpha_{tt'} & \text{if } |t-t'| \leq m \\ 0 & \text{if } |t-t'| > m \end{cases}$$

If (`corstr = "non_stat_M_dep"`), you must also specify  $Mv = m$ , where  $m$  is the number of periods  $t$  of dependence. This option relaxes the assumption that the correlations are the same for all observations of the same  $|t - t'|$  periods apart.

Sample “working” correlation for Non-stationary 2 dependence ( $Mv=2$ )

$$\begin{pmatrix} 1 & \alpha_{12} & \alpha_{13} & 0 & 0 \\ \alpha_{12} & 1 & \alpha_{23} & \alpha_{24} & 0 \\ \alpha_{13} & \alpha_{23} & 1 & \alpha_{34} & \alpha_{35} \\ 0 & \alpha_{24} & \alpha_{34} & 1 & \alpha_{45} \\ 0 & 0 & \alpha_{35} & \alpha_{45} & 1 \end{pmatrix}$$

- Exchangeable (`corstr = "exchangeable"`):  $\text{cor}(y_{it}, y_{it'}) = \alpha, \forall t, t'$  with  $t \neq t'$ . Choose this option if the correlations are assumed to be the same for all observations within the cluster.

Sample “working” correlation for Exchangeable

$$\begin{pmatrix} 1 & \alpha & \alpha & \alpha & \alpha \\ \alpha & 1 & \alpha & \alpha & \alpha \\ \alpha & \alpha & 1 & \alpha & \alpha \\ \alpha & \alpha & \alpha & 1 & \alpha \\ \alpha & \alpha & \alpha & \alpha & 1 \end{pmatrix}$$

- Stationary  $m$ th order autoregressive (**corstr** = "AR-M"): If (**corstr** = "AR-M"), you must also specify **Mv** =  $m$ , where  $m$  is the number of periods  $t$  of dependence. For example, the first order autoregressive model (AR-1) implies  $\text{cor}(y_{it}, y_{it'}) = \alpha^{|t-t'|}, \forall t, t'$  with  $t \neq t'$ . In AR-1, observation 1 and observation 2 have a correlation of  $\alpha$ . Observation 2 and observation 3 also have a correlation of  $\alpha$ . Observation 1 and observation 3 have a correlation of  $\alpha^2$ , which is a function of how 1 and 2 are correlated ( $\alpha$ ) multiplied by how 2 and 3 are correlated ( $\alpha$ ). Observation 1 and 4 have a correlation that is a function of the correlation between 1 and 2, 2 and 3, and 3 and 4, and so forth.

Sample “working” correlation for Stationary AR-1 (**Mv**=1)

$$\begin{pmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ \alpha & 1 & \alpha & \alpha^2 & \alpha^3 \\ \alpha^2 & \alpha & 1 & \alpha & \alpha^2 \\ \alpha^3 & \alpha^2 & \alpha & 1 & \alpha \\ \alpha^4 & \alpha^3 & \alpha^2 & \alpha & 1 \end{pmatrix}$$

- Unstructured (**corstr** = "unstructured"):  $\text{cor}(y_{it}, y_{it'}) = \alpha_{tt'}, \forall t, t'$  with  $t \neq t'$ . No constraints are placed on the correlations, which are then estimated from the data.
- **Mv**: defaults to 1. It specifies the number of periods of correlation and only needs to be specified when **corstr** is "stat\_M\_dep", "non\_stat\_M\_dep", or "AR-M".
- **R**: defaults to NULL. It specifies a user-defined correlation matrix rather than estimating it from the data. The argument is used only when **corstr** is "fixed". The input is a  $T \times T$  matrix of correlations, where  $T$  is the size of the largest cluster.

### 3.0.15 Examples

#### 1. Example with AR-1 Dependence

Attaching the sample turnout dataset:

```
> data(macro)
```

Estimating model and presenting summary:

```
> z.out <- zelig(unem ~ gdp + capmob + trade, model = "normal.gee", id = "country", dat
```

(Intercept)		gdp	capmob	trade
6.18129445	-0.32360059	1.42193926	0.01985421	

How to cite this model in Zelig:

Patrick Lam. 2013.

"normal.gee: General Estimating Equation for Normal Regression"

in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software,"

<http://gking.harvard.edu/zelig>

> summary(z.out)

GEE: GENERALIZED LINEAR MODELS FOR DEPENDENT DATA  
gee S-function, version 4.13 modified 98/01/27 (1998)

Model:

Link:	Identity
Variance to Mean Relation:	Gaussian
Correlation Structure:	AR-M , M = 1

Call:

gee(formula = formula, id = id, corstr = corstr, family = gaussian(),  
R = R, data = data, Mv = 1, .hook = robust.gee.hook)

Summary of Residuals:

	Min	1Q	Median	3Q	Max
	-3.8177474	-1.9082221	0.1124818	2.8287233	8.0261275

Coefficients:

	Estimate	Naive S.E.	Naive z	Robust S.E.	Robust z
(Intercept)	3.96818567	0.912532716	4.348541	0.77474335	5.121936
gdp	-0.06553926	0.016222536	-4.040013	0.01737177	-3.772744
capmob	0.33382352	0.125003704	2.670509	0.16212556	2.059043
trade	0.01616045	0.009822323	1.645278	0.01102485	1.465820

Estimated Scale Parameter: 9.512685

Number of Iterations: 4

Working Correlation

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]
[1,]	1.0000000	0.9701417	0.9411749	0.9130731	0.8858103	0.8593615	0.8337024
[2,]	0.9701417	1.0000000	0.9701417	0.9411749	0.9130731	0.8858103	0.8593615
[3,]	0.9411749	0.9701417	1.0000000	0.9701417	0.9411749	0.9130731	0.8858103
[4,]	0.9130731	0.9411749	0.9701417	1.0000000	0.9701417	0.9411749	0.9130731
[5,]	0.8858103	0.9130731	0.9411749	0.9701417	1.0000000	0.9701417	0.9411749

[6,]	0.8593615	0.8858103	0.9130731	0.9411749	0.9701417	1.0000000	0.9701417
[7,]	0.8337024	0.8593615	0.8858103	0.9130731	0.9411749	0.9701417	1.0000000
[8,]	0.8088095	0.8337024	0.8593615	0.8858103	0.9130731	0.9411749	0.9701417
[9,]	0.7846598	0.8088095	0.8337024	0.8593615	0.8858103	0.9130731	0.9411749
[10,]	0.7612312	0.7846598	0.8088095	0.8337024	0.8593615	0.8858103	0.9130731
[11,]	0.7385021	0.7612312	0.7846598	0.8088095	0.8337024	0.8593615	0.8858103
[12,]	0.7164517	0.7385021	0.7612312	0.7846598	0.8088095	0.8337024	0.8593615
[13,]	0.6950597	0.7164517	0.7385021	0.7612312	0.7846598	0.8088095	0.8337024
[14,]	0.6743064	0.6950597	0.7164517	0.7385021	0.7612312	0.7846598	0.8088095
[15,]	0.6541728	0.6743064	0.6950597	0.7164517	0.7385021	0.7612312	0.7846598
[16,]	0.6346403	0.6541728	0.6743064	0.6950597	0.7164517	0.7385021	0.7612312
[17,]	0.6156910	0.6346403	0.6541728	0.6743064	0.6950597	0.7164517	0.7385021
[18,]	0.5973075	0.6156910	0.6346403	0.6541728	0.6743064	0.6950597	0.7164517
[19,]	0.5794730	0.5973075	0.6156910	0.6346403	0.6541728	0.6743064	0.6950597
[20,]	0.5621709	0.5794730	0.5973075	0.6156910	0.6346403	0.6541728	0.6743064
[21,]	0.5453854	0.5621709	0.5794730	0.5973075	0.6156910	0.6346403	0.6541728
[22,]	0.5291011	0.5453854	0.5621709	0.5794730	0.5973075	0.6156910	0.6346403
[23,]	0.5133031	0.5291011	0.5453854	0.5621709	0.5794730	0.5973075	0.6156910
[24,]	0.4979767	0.5133031	0.5291011	0.5453854	0.5621709	0.5794730	0.5973075
[25,]	0.4831080	0.4979767	0.5133031	0.5291011	0.5453854	0.5621709	0.5794730
[ ,8] [ ,9] [ ,10] [ ,11] [ ,12] [ ,13] [ ,14]							
[1,]	0.8088095	0.7846598	0.7612312	0.7385021	0.7164517	0.6950597	0.6743064
[2,]	0.8337024	0.8088095	0.7846598	0.7612312	0.7385021	0.7164517	0.6950597
[3,]	0.8593615	0.8337024	0.8088095	0.7846598	0.7612312	0.7385021	0.7164517
[4,]	0.8858103	0.8593615	0.8337024	0.8088095	0.7846598	0.7612312	0.7385021
[5,]	0.9130731	0.8858103	0.8593615	0.8337024	0.8088095	0.7846598	0.7612312
[6,]	0.9411749	0.9130731	0.8858103	0.8593615	0.8337024	0.8088095	0.7846598
[7,]	0.9701417	0.9411749	0.9130731	0.8858103	0.8593615	0.8337024	0.8088095
[8,]	1.0000000	0.9701417	0.9411749	0.9130731	0.8858103	0.8593615	0.8337024
[9,]	0.9701417	1.0000000	0.9701417	0.9411749	0.9130731	0.8858103	0.8593615
[10,]	0.9411749	0.9701417	1.0000000	0.9701417	0.9411749	0.9130731	0.8858103
[11,]	0.9130731	0.9411749	0.9701417	1.0000000	0.9701417	0.9411749	0.9130731
[12,]	0.8858103	0.9130731	0.9411749	0.9701417	1.0000000	0.9701417	0.9411749
[13,]	0.8593615	0.8858103	0.9130731	0.9411749	0.9701417	1.0000000	0.9701417
[14,]	0.8337024	0.8593615	0.8858103	0.9130731	0.9411749	0.9701417	1.0000000
[15,]	0.8088095	0.8337024	0.8593615	0.8858103	0.9130731	0.9411749	0.9701417
[16,]	0.7846598	0.8088095	0.8337024	0.8593615	0.8858103	0.9130731	0.9411749
[17,]	0.7612312	0.7846598	0.8088095	0.8337024	0.8593615	0.8858103	0.9130731
[18,]	0.7385021	0.7612312	0.7846598	0.8088095	0.8337024	0.8593615	0.8858103
[19,]	0.7164517	0.7385021	0.7612312	0.7846598	0.8088095	0.8337024	0.8593615
[20,]	0.6950597	0.7164517	0.7385021	0.7612312	0.7846598	0.8088095	0.8337024
[21,]	0.6743064	0.6950597	0.7164517	0.7385021	0.7612312	0.7846598	0.8088095
[22,]	0.6541728	0.6743064	0.6950597	0.7164517	0.7385021	0.7612312	0.7846598
[23,]	0.6346403	0.6541728	0.6743064	0.6950597	0.7164517	0.7385021	0.7612312
[24,]	0.6156910	0.6346403	0.6541728	0.6743064	0.6950597	0.7164517	0.7385021
[25,]	0.5973075	0.6156910	0.6346403	0.6541728	0.6743064	0.6950597	0.7164517



	[,15]	[,16]	[,17]	[,18]	[,19]	[,20]	[,21]
[1,]	0.6541728	0.6346403	0.6156910	0.5973075	0.5794730	0.5621709	0.5453854
[2,]	0.6743064	0.6541728	0.6346403	0.6156910	0.5973075	0.5794730	0.5621709
[3,]	0.6950597	0.6743064	0.6541728	0.6346403	0.6156910	0.5973075	0.5794730
[4,]	0.7164517	0.6950597	0.6743064	0.6541728	0.6346403	0.6156910	0.5973075
[5,]	0.7385021	0.7164517	0.6950597	0.6743064	0.6541728	0.6346403	0.6156910
[6,]	0.7612312	0.7385021	0.7164517	0.6950597	0.6743064	0.6541728	0.6346403
[7,]	0.7846598	0.7612312	0.7385021	0.7164517	0.6950597	0.6743064	0.6541728
[8,]	0.8088095	0.7846598	0.7612312	0.7385021	0.7164517	0.6950597	0.6743064
[9,]	0.8337024	0.8088095	0.7846598	0.7612312	0.7385021	0.7164517	0.6950597
[10,]	0.8593615	0.8337024	0.8088095	0.7846598	0.7612312	0.7385021	0.7164517
[11,]	0.8858103	0.8593615	0.8337024	0.8088095	0.7846598	0.7612312	0.7385021
[12,]	0.9130731	0.8858103	0.8593615	0.8337024	0.8088095	0.7846598	0.7612312
[13,]	0.9411749	0.9130731	0.8858103	0.8593615	0.8337024	0.8088095	0.7846598
[14,]	0.9701417	0.9411749	0.9130731	0.8858103	0.8593615	0.8337024	0.8088095
[15,]	1.0000000	0.9701417	0.9411749	0.9130731	0.8858103	0.8593615	0.8337024
[16,]	0.9701417	1.0000000	0.9701417	0.9411749	0.9130731	0.8858103	0.8593615
[17,]	0.9411749	0.9701417	1.0000000	0.9701417	0.9411749	0.9130731	0.8858103
[18,]	0.9130731	0.9411749	0.9701417	1.0000000	0.9701417	0.9411749	0.9130731
[19,]	0.8858103	0.9130731	0.9411749	0.9701417	1.0000000	0.9701417	0.9411749
[20,]	0.8593615	0.8858103	0.9130731	0.9411749	0.9701417	1.0000000	0.9701417
[21,]	0.8337024	0.8593615	0.8858103	0.9130731	0.9411749	0.9701417	1.0000000
[22,]	0.8088095	0.8337024	0.8593615	0.8858103	0.9130731	0.9411749	0.9701417
[23,]	0.7846598	0.8088095	0.8337024	0.8593615	0.8858103	0.9130731	0.9411749
[24,]	0.7612312	0.7846598	0.8088095	0.8337024	0.8593615	0.8858103	0.9130731
[25,]	0.7385021	0.7612312	0.7846598	0.8088095	0.8337024	0.8593615	0.8858103

	[,22]	[,23]	[,24]	[,25]
[1,]	0.5291011	0.5133031	0.4979767	0.4831080
[2,]	0.5453854	0.5291011	0.5133031	0.4979767
[3,]	0.5621709	0.5453854	0.5291011	0.5133031
[4,]	0.5794730	0.5621709	0.5453854	0.5291011
[5,]	0.5973075	0.5794730	0.5621709	0.5453854
[6,]	0.6156910	0.5973075	0.5794730	0.5621709
[7,]	0.6346403	0.6156910	0.5973075	0.5794730
[8,]	0.6541728	0.6346403	0.6156910	0.5973075
[9,]	0.6743064	0.6541728	0.6346403	0.6156910
[10,]	0.6950597	0.6743064	0.6541728	0.6346403
[11,]	0.7164517	0.6950597	0.6743064	0.6541728
[12,]	0.7385021	0.7164517	0.6950597	0.6743064
[13,]	0.7612312	0.7385021	0.7164517	0.6950597
[14,]	0.7846598	0.7612312	0.7385021	0.7164517
[15,]	0.8088095	0.7846598	0.7612312	0.7385021
[16,]	0.8337024	0.8088095	0.7846598	0.7612312
[17,]	0.8593615	0.8337024	0.8088095	0.7846598
[18,]	0.8858103	0.8593615	0.8337024	0.8088095
[19,]	0.9130731	0.8858103	0.8593615	0.8337024

```
[20,] 0.9411749 0.9130731 0.8858103 0.8593615
[21,] 0.9701417 0.9411749 0.9130731 0.8858103
[22,] 1.0000000 0.9701417 0.9411749 0.9130731
[23,] 0.9701417 1.0000000 0.9701417 0.9411749
[24,] 0.9411749 0.9701417 1.0000000 0.9701417
[25,] 0.9130731 0.9411749 0.9701417 1.0000000
```

Set explanatory variables to their default (mean/mode) values, with high (80th percentile) and low (20th percentile) values:

```
> x.high <- setx(z.out, trade = quantile(macro$trade, 0.8))
> x.low <- setx(z.out, trade = quantile(macro$trade, 0.2))
```

Generate first differences for the effect of high versus low trade on GDP:

```
> s.out <- sim(z.out, x = x.high, x1 = x.low)
> summary(s.out)
```

Model: normal.gee

Number of simulations: 1000

Values of X

```
(Intercept)      gdp      capmob      trade
1           1 3.254223 -0.8914286 79.10131
attr("assign")
[1] 0 1 2 3
```

Values of X1

```
(Intercept)      gdp      capmob      trade
1           1 3.254223 -0.8914286 37.29106
attr("assign")
[1] 0 1 2 3
```

Expected Values (for x): E(Y|X)

```
mean    sd    50%   2.5%  97.5%
4.726 0.524 4.711 3.757  5.78
```

Expected Values (for x1): E(Y|X1)

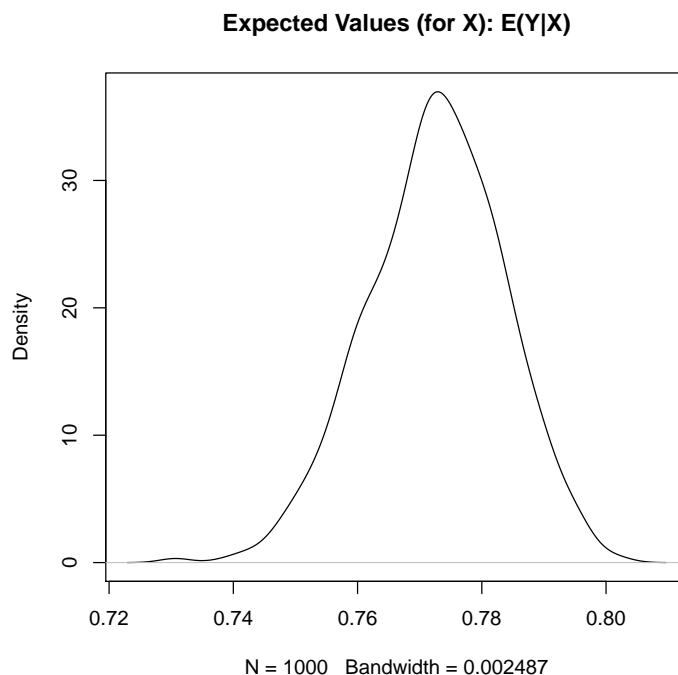
```
mean    sd    50%   2.5%  97.5%
4.083 0.513 4.094 3.056  5.095
```

First Differences: E(Y|X1) - E(Y|X)

```
mean    sd    50%   2.5%  97.5%
-0.642 0.466 -0.638 -1.612  0.272
```

Generate a plot of quantities of interest:

```
> plot(s.out)
```



### 3.0.16 The Model

Suppose we have a panel dataset, with  $Y_{it}$  denoting the continuous dependent variable for unit  $i$  at time  $t$ .  $Y_i$  is a vector or cluster of correlated data where  $y_{it}$  is correlated with  $y_{it'}$  for some or all  $t, t'$ . Note that the model assumes correlations within  $i$  but independence across  $i$ .

- The *stochastic component* is given by the joint and marginal distributions

$$\begin{aligned} Y_i &\sim f(y_i | \mu_i) \\ Y_{it} &\sim g(y_{it} | \mu_{it}) \end{aligned}$$

where  $f$  and  $g$  are unspecified distributions with means  $\mu_i$  and  $\mu_{it}$ . GEE models make no distributional assumptions and only require three specifications: a mean function, a variance function, and a correlation structure.

- The *systematic component* is the *mean function*, given by:

$$\mu_{it} = x_{it}\beta$$

where  $x_{it}$  is the vector of  $k$  explanatory variables for unit  $i$  at time  $t$  and  $\beta$  is the vector of coefficients.

- The *variance function* is given by:

$$V_{it} = 1$$

- The *correlation structure* is defined by a  $T \times T$  “working” correlation matrix, where  $T$  is the size of the largest cluster. Users must specify the structure of the “working” correlation matrix *a priori*. The “working” correlation matrix then enters the variance term for each  $i$ , given by:

$$V_i = \phi A_i^{\frac{1}{2}} R_i(\alpha) A_i^{\frac{1}{2}}$$

where  $A_i$  is a  $T \times T$  diagonal matrix with the variance function  $V_{it} = 1$  as the  $t$ th diagonal element (in the case of GEE normal,  $A_i$  is the identity matrix),  $R_i(\alpha)$  is the “working” correlation matrix, and  $\phi$  is a scale parameter. The parameters are then estimated via a quasi-likelihood approach.

- In GEE models, if the mean is correctly specified, but the variance and correlation structure are incorrectly specified, then GEE models provide consistent estimates of the parameters and thus the mean function as well, while consistent estimates of the standard errors can be obtained via a robust “sandwich” estimator. Similarly, if the mean and variance are correctly specified but the correlation structure is incorrectly specified, the parameters can be estimated consistently and the standard errors can be estimated consistently with the sandwich estimator. If all three are specified correctly, then the estimates of the parameters are more efficient.
- The robust “sandwich” estimator gives consistent estimates of the standard errors when the correlations are specified incorrectly only if the number of units  $i$  is relatively large and the number of repeated periods  $t$  is relatively small. Otherwise, one should use the “naïve” model-based standard errors, which assume that the specified correlations are close approximations to the true underlying correlations. See [?] for more details.

### 3.0.17 Quantities of Interest

- All quantities of interest are for marginal means rather than joint means.
- The method of bootstrapping generally should not be used in GEE models. If you must bootstrap, bootstrapping should be done within clusters, which is not currently supported in Zelig. For conditional prediction models, data should be matched within clusters.
- The expected values (`qi$ev`) for the GEE normal model is the mean of simulations from the stochastic component:

$$E(Y) = \mu_c = x_c \beta,$$

given draws of  $\beta$  from its sampling distribution, where  $x_c$  is a vector of values, one for each independent variable, chosen by the user.

- The first difference (`qi$fd`) for the GEE normal model is defined as

$$FD = \Pr(Y = 1 \mid x_1) - \Pr(Y = 1 \mid x).$$

- In conditional prediction models, the average expected treatment effect (`att.ev`) for the treatment group is

$$\frac{1}{\sum_{i=1}^n \sum_{t=1}^T tr_{it}} \sum_{i:tr_{it}=1}^n \sum_{t:tr_{it}=1}^T \{Y_{it}(tr_{it} = 1) - E[Y_{it}(tr_{it} = 0)]\},$$

where  $tr_{it}$  is a binary explanatory variable defining the treatment ( $tr_{it} = 1$ ) and control ( $tr_{it} = 0$ ) groups. Variation in the simulations are due to uncertainty in simulating  $E[Y_{it}(tr_{it} = 0)]$ , the counterfactual expected value of  $Y_{it}$  for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to  $tr_{it} = 0$ .

### 3.0.18 Output Values

The output of each `Zelig` command contains useful information which you may view. For example, if you run `z.out <- zelig(y ~ x, model = "normal.gee", id, data)`, then you may examine the available information in `z.out` by using `names(z.out)`, see the `coefficients` by using `z.out$coefficients`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output object `z.out`, you may extract:
  - `coefficients`: parameter estimates for the explanatory variables.
  - `residuals`: the working residuals in the final iteration of the fit.
  - `fitted.values`: the vector of fitted values for the systemic component,  $\mu_{it}$ .
  - `linear.predictors`: the vector of  $x_{it}\beta$
  - `max.id`: the size of the largest cluster.
- From `summary(z.out)`, you may extract:
  - `coefficients`: the parameter estimates with their associated standard errors,  $p$ -values, and  $z$ -statistics.
  - `working.correlation`: the “working” correlation matrix
- From the `sim()` output object `s.out`, you may extract quantities of interest arranged as matrices indexed by simulation  $\times$   $x$ -observation (for more than one  $x$ -observation). Available quantities are:
  - `qi$ev`: the simulated expected values for the specified values of  $x$ .

- `qi$fd`: the simulated first difference in the expected probabilities for the values specified in `x` and `x1`.
- `qi$att.ev`: the simulated average expected treatment effect for the treated from conditional prediction models.

## How To Cite the *normal.gee* Zelig model

Patrick Lam. 2007. “normal.gee: Generalized Estimating Equation for Normal Regression,” in Kosuke Imai, Gary King, and Olivia Lau, “Zelig: Everyone’s Statistical Software,” <http://gking.harvard.edu/zelig>.

## How to Cite the Zelig Software Package

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Imai, Kosuke, Gary King, and Olivia Lau. (2008). “Toward A Common Framework for Statistical Analysis and Development.” *Journal of Computational and Graphical Statistics*, Vol. 17, No. 4 (December), pp. 892-913.

## 4 poisson.gee: Generalized Estimating Equation for Poisson Regression

The GEE poisson estimates the same model as the standard poisson regression (appropriate when your dependent variable represents the number of independent events that occur during a fixed period of time). Unlike in poisson regression, GEE poisson allows for dependence within clusters, such as in longitudinal data, although its use is not limited to just panel data. The user must first specify a “working” correlation matrix for the clusters, which models the dependence of each observation with other observations in the same cluster. The “working” correlation matrix is a  $T \times T$  matrix of correlations, where  $T$  is the size of the largest cluster and the elements of the matrix are correlations between within-cluster observations. The appeal of GEE models is that it gives consistent estimates of the parameters and consistent estimates of the standard errors can be obtained using a robust “sandwich” estimator even if the “working” correlation matrix is incorrectly specified. If the “working” correlation matrix is correctly specified, GEE models will give more efficient estimates of the parameters. GEE models measure population-averaged effects as opposed to cluster-specific effects (See (author?) [4]).

#### 4.0.19 Syntax

```
> z.out <- zelig(Y ~ X1 + X2, model = "poisson.gee",
               id = "X3", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

where `id` is a variable which identifies the clusters. The data should be sorted by `id` and should be ordered within each cluster when appropriate.

#### 4.0.20 Additional Inputs

- **robust**: defaults to `TRUE`. If `TRUE`, consistent standard errors are estimated using a “sandwich” estimator.

Use the following arguments to specify the structure of the “working” correlations within clusters:

- **corstr**: defaults to `"independence"`. It can take on the following arguments:
  - Independence (**corstr** = `"independence"`):  $\text{cor}(y_{it}, y_{it'}) = 0, \forall t, t'$  with  $t \neq t'$ . It assumes that there is no correlation within the clusters and the model becomes equivalent to standard poisson regression. The “working” correlation matrix is the identity matrix.
  - Fixed (**corstr** = `"fixed"`): If selected, the user must define the “working” correlation matrix with the `R` argument rather than estimating it from the model.
  - Stationary  $m$  dependent (**corstr** = `"stat_M_dep"`):

$$\text{cor}(y_{it}, y_{it'}) = \begin{cases} \alpha_{|t-t'|} & \text{if } |t-t'| \leq m \\ 0 & \text{if } |t-t'| > m \end{cases}$$

If (**corstr** = `"stat_M_dep"`), you must also specify `Mv = m`, where  $m$  is the number of periods  $t$  of dependence. Choose this option when the correlations are assumed to be the same for observations of the same  $|t - t'|$  periods apart for  $|t - t'| \leq m$ .

Sample “working” correlation for Stationary 2 dependence (`Mv=2`)

$$\begin{pmatrix} 1 & \alpha_1 & \alpha_2 & 0 & 0 \\ \alpha_1 & 1 & \alpha_1 & \alpha_2 & 0 \\ \alpha_2 & \alpha_1 & 1 & \alpha_1 & \alpha_2 \\ 0 & \alpha_2 & \alpha_1 & 1 & \alpha_1 \\ 0 & 0 & \alpha_2 & \alpha_1 & 1 \end{pmatrix}$$

- Non-stationary  $m$  dependent (`corstr` = "non\_stat\_M\_dep"):

$$\text{cor}(y_{it}, y_{it'}) = \begin{cases} \alpha_{tt'} & \text{if } |t - t'| \leq m \\ 0 & \text{if } |t - t'| > m \end{cases}$$

If (`corstr` = "non\_stat\_M\_dep"), you must also specify  $Mv = m$ , where  $m$  is the number of periods  $t$  of dependence. This option relaxes the assumption that the correlations are the same for all observations of the same  $|t - t'|$  periods apart.

Sample “working” correlation for Non-stationary 2 dependence  
( $Mv=2$ )

$$\begin{pmatrix} 1 & \alpha_{12} & \alpha_{13} & 0 & 0 \\ \alpha_{12} & 1 & \alpha_{23} & \alpha_{24} & 0 \\ \alpha_{13} & \alpha_{23} & 1 & \alpha_{34} & \alpha_{35} \\ 0 & \alpha_{24} & \alpha_{34} & 1 & \alpha_{45} \\ 0 & 0 & \alpha_{35} & \alpha_{45} & 1 \end{pmatrix}$$

- Exchangeable (`corstr` = "exchangeable"):  $\text{cor}(y_{it}, y_{it'}) = \alpha, \forall t, t'$  with  $t \neq t'$ . Choose this option if the correlations are assumed to be the same for all observations within the cluster.

Sample “working” correlation for Exchangeable

$$\begin{pmatrix} 1 & \alpha & \alpha & \alpha & \alpha \\ \alpha & 1 & \alpha & \alpha & \alpha \\ \alpha & \alpha & 1 & \alpha & \alpha \\ \alpha & \alpha & \alpha & 1 & \alpha \\ \alpha & \alpha & \alpha & \alpha & 1 \end{pmatrix}$$

- Stationary  $m$ th order autoregressive (`corstr` = "AR-M"): If (`corstr` = "AR-M"), you must also specify  $Mv = m$ , where  $m$  is the number of periods  $t$  of dependence. For example, the first order autoregressive model (AR-1) implies  $\text{cor}(y_{it}, y_{it'}) = \alpha^{|t-t'|}, \forall t, t'$  with  $t \neq t'$ . In AR-1, observation 1 and observation 2 have a correlation of  $\alpha$ . Observation 2 and observation 3 also have a correlation of  $\alpha$ . Observation 1 and observation 3 have a correlation of  $\alpha^2$ , which is a function of how 1 and 2 are correlated ( $\alpha$ ) multiplied by how 2 and 3 are correlated ( $\alpha$ ). Observation 1 and 4 have a correlation that is a function of the correlation between 1 and 2, 2 and 3, and 3 and 4, and so forth.

Sample “working” correlation for Stationary AR-1 ( $Mv=1$ )

$$\begin{pmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ \alpha & 1 & \alpha & \alpha^2 & \alpha^3 \\ \alpha^2 & \alpha & 1 & \alpha & \alpha^2 \\ \alpha^3 & \alpha^2 & \alpha & 1 & \alpha \\ \alpha^4 & \alpha^3 & \alpha^2 & \alpha & 1 \end{pmatrix}$$



- Unstructured (`corstr = "unstructured"`):  $\text{cor}(y_{it}, y_{it'}) = \alpha_{tt'}, \forall t, t'$  with  $t \neq t'$ . No constraints are placed on the correlations, which are then estimated from the data.
- `Mv`: defaults to 1. It specifies the number of periods of correlation and only needs to be specified when `corstr` is `"stat_M_dep"`, `"non_stat_M_dep"`, or `"AR-M"`.
- `R`: defaults to `NULL`. It specifies a user-defined correlation matrix rather than estimating it from the data. The argument is used only when `corstr` is `"fixed"`. The input is a  $T \times T$  matrix of correlations, where  $T$  is the size of the largest cluster.

#### 4.0.21 Examples

##### 1. Example with Exchangeable Dependence

Attaching the sample turnout dataset:

```
> data(sanction)
```

Variable identifying clusters

```
> sanction$cluster <- c(rep(c(1:15),5),rep(c(16),3))
```

Sorting by cluster

```
> sorted.sanction <- sanction[order(sanction$cluster),]
```

Estimating model and presenting summary:

```
> z.out <- zelig(num ~ target + coop, model = "poisson.gee", id = "cluster", data = sorted.sanction)
```

```
(Intercept)      target      coop
-0.96771994 -0.02102351  1.21081908
```

How to cite this model in Zelig:

Patrick Lam. 2013.

"poisson.gee: General Estimating Equation for Poisson Regression"

in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software,"

<http://gking.harvard.edu/zelig>

```
> summary(z.out)
```

```
GEE:  GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)
```

Model:

Link:                                      Logarithm

Variance to Mean Relation: Poisson  
Correlation Structure: Exchangeable

Call:  
gee(formula = formula, id = id, corstr = corstr, family = poisson(),  
data = data, R = R, .hook = robust.gee.hook)

Summary of Residuals:

	Min	1Q	Median	3Q	Max
	-39.1944672	-2.1913793	-0.2236836	-0.2047618	106.5134108

Coefficients:

	Estimate	Naive S.E.	Naive z	Robust S.E.	Robust z
(Intercept)	-0.96680028	0.7196563	-1.3434195	0.4550626	-2.12454341
target	-0.02083277	0.2413024	-0.0863347	0.3341394	-0.06234754
coop	1.21033147	0.1914327	6.3224910	0.2631640	4.59915320

Estimated Scale Parameter: 17.10881  
Number of Iterations: 2

Working Correlation

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	1.00000000	-0.01180279	-0.01180279	-0.01180279	-0.01180279
[2,]	-0.01180279	1.00000000	-0.01180279	-0.01180279	-0.01180279
[3,]	-0.01180279	-0.01180279	1.00000000	-0.01180279	-0.01180279
[4,]	-0.01180279	-0.01180279	-0.01180279	1.00000000	-0.01180279
[5,]	-0.01180279	-0.01180279	-0.01180279	-0.01180279	1.00000000

Set explanatory variables to their default values:

```
> x.out <- setx(z.out)
```

Simulate quantities of interest

```
> s.out <- sim(z.out, x = x.out)
> summary(s.out)
```

Model: poisson.gee  
Number of simulations: 1000

Values of X

	(Intercept)	target	coop
1	1	2.141026	1.807692

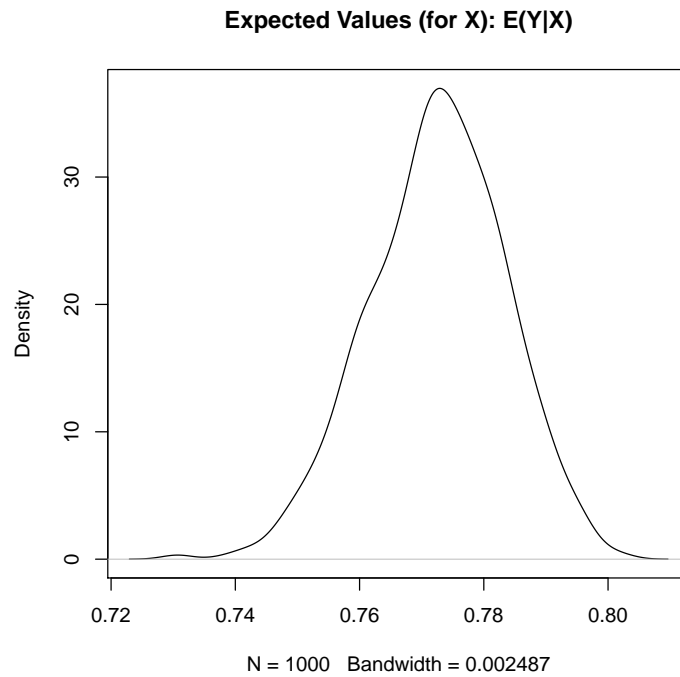
```
attr("assign")
[1] 0 1 2
```

Expected Values (for x): E(Y|X)

mean	sd	50%	2.5%	97.5%
0.891	0.216	0.846	0.613	1.409

Generate a plot of quantities of interest:

```
> plot(s.out)
```



#### 4.0.22 The Model

Suppose we have a panel dataset, with  $Y_{it}$  denoting the dependent variable of the number of independent events for a fixed period of time for unit  $i$  at time  $t$ .  $Y_i$  is a vector or cluster of correlated data where  $y_{it}$  is correlated with  $y_{it'}$  for some or all  $t, t'$ . Note that the model assumes correlations within  $i$  but independence across  $i$ .

- The *stochastic component* is given by the joint and marginal distributions

$$\begin{aligned} Y_i &\sim f(y_i | \lambda_i) \\ Y_{it} &\sim g(y_{it} | \lambda_{it}) \end{aligned}$$

where  $f$  and  $g$  are unspecified distributions with means  $\lambda_i$  and  $\lambda_{it}$ . GEE models make no distributional assumptions and only require three specifications: a mean function, a variance function, and a correlation structure.

- The *systematic component* is the *mean function*, given by:

$$\lambda_{it} = \exp(x_{it}\beta)$$

where  $x_{it}$  is the vector of  $k$  explanatory variables for unit  $i$  at time  $t$  and  $\beta$  is the vector of coefficients.

- The *variance function* is given by:

$$V_{it} = \lambda_{it}$$

- The *correlation structure* is defined by a  $T \times T$  “working” correlation matrix, where  $T$  is the size of the largest cluster. Users must specify the structure of the “working” correlation matrix *a priori*. The “working” correlation matrix then enters the variance term for each  $i$ , given by:

$$V_i = \phi A_i^{\frac{1}{2}} R_i(\alpha) A_i^{\frac{1}{2}}$$

where  $A_i$  is a  $T \times T$  diagonal matrix with the variance function  $V_{it} = \lambda_{it}$  as the  $t$ th diagonal element,  $R_i(\alpha)$  is the “working” correlation matrix, and  $\phi$  is a scale parameter. The parameters are then estimated via a quasi-likelihood approach.

- In GEE models, if the mean is correctly specified, but the variance and correlation structure are incorrectly specified, then GEE models provide consistent estimates of the parameters and thus the mean function as well, while consistent estimates of the standard errors can be obtained via a robust “sandwich” estimator. Similarly, if the mean and variance are correctly specified but the correlation structure is incorrectly specified, the parameters can be estimated consistently and the standard errors can be estimated consistently with the sandwich estimator. If all three are specified correctly, then the estimates of the parameters are more efficient.
- The robust “sandwich” estimator gives consistent estimates of the standard errors when the correlations are specified incorrectly only if the number of units  $i$  is relatively large and the number of repeated periods  $t$  is relatively small. Otherwise, one should use the “naïve” model-based standard errors, which assume that the specified correlations are close approximations to the true underlying correlations. See [?] for more details.

#### 4.0.23 Quantities of Interest

- All quantities of interest are for marginal means rather than joint means.
- The method of bootstrapping generally should not be used in GEE models. If you must bootstrap, bootstrapping should be done within clusters, which is not currently supported in Zelig. For conditional prediction models, data should be matched within clusters.

- The expected values (`qi$ev`) for the GEE poisson model is the mean of simulations from the stochastic component:

$$E(Y) = \lambda_c = \exp(x_c \beta),$$

given draws of  $\beta$  from its sampling distribution, where  $x_c$  is a vector of values, one for each independent variable, chosen by the user.

- The first difference (`qi$fd`) for the GEE poisson model is defined as

$$FD = \Pr(Y = 1 \mid x_1) - \Pr(Y = 1 \mid x).$$

- In conditional prediction models, the average expected treatment effect (`att.ev`) for the treatment group is

$$\frac{1}{\sum_{i=1}^n \sum_{t=1}^T tr_{it}} \sum_{i:tr_{it}=1}^n \sum_{t:tr_{it}=1}^T \{Y_{it}(tr_{it} = 1) - E[Y_{it}(tr_{it} = 0)]\},$$

where  $tr_{it}$  is a binary explanatory variable defining the treatment ( $tr_{it} = 1$ ) and control ( $tr_{it} = 0$ ) groups. Variation in the simulations are due to uncertainty in simulating  $E[Y_{it}(tr_{it} = 0)]$ , the counterfactual expected value of  $Y_{it}$  for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to  $tr_{it} = 0$ .

#### 4.0.24 Output Values

The output of each `Zelig` command contains useful information which you may view. For example, if you run `z.out <- zelig(y ~ x, model = "poisson.gee", id, data)`, then you may examine the available information in `z.out` by using `names(z.out)`, see the `coefficients` by using `z.out$coefficients`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output object `z.out`, you may extract:
  - `coefficients`: parameter estimates for the explanatory variables.
  - `residuals`: the working residuals in the final iteration of the fit.
  - `fitted.values`: the vector of fitted values for the systemic component,  $\lambda_{it}$ .
  - `linear.predictors`: the vector of  $x_{it}\beta$
  - `max.id`: the size of the largest cluster.
- From `summary(z.out)`, you may extract:
  - `coefficients`: the parameter estimates with their associated standard errors,  $p$ -values, and  $z$ -statistics.

- `working.correlation`: the “working” correlation matrix
- From the `sim()` output object `s.out`, you may extract quantities of interest arranged as matrices indexed by simulation  $\times$  `x`-observation (for more than one `x`-observation). Available quantities are:
  - `qi$ev`: the simulated expected values for the specified values of `x`.
  - `qi$fd`: the simulated first difference in the expected probabilities for the values specified in `x` and `x1`.
  - `qi$att.ev`: the simulated average expected treatment effect for the treated from conditional prediction models.

## How To Cite *poisson.gee* Zelig model

Patrick Lam. 2007. “*poisson.gee*: Generalized Estimating Equation for Poisson Regression,” in Kosuke Imai, Gary King, and Olivia Lau, “Zelig: Everyone’s Statistical Software,” <http://gking.harvard.edu/zelig>.

## How to Cite the Zelig Software Package

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Imai, Kosuke, Gary King, and Olivia Lau. (2008). “Toward A Common Framework for Statistical Analysis and Development.” *Journal of Computational and Graphical Statistics*, Vol. 17, No. 4 (December), pp. 892-913.

## See also

The `gee` function is part of the `gee` package by Vincent J. Carey, ported to R by Thomas Lumley and Brian Ripley. Advanced users may wish to refer to `help(gee)` and `help(family)`. Sample data are from [3]. Please inquire with Lisa Martin before publishing results from these data, as this dataset includes errors that have since been corrected.

## 5 `probit.gee`: Generalized Estimating Equation for Probit Regression

The GEE probit estimates the same model as the standard probit regression (appropriate when you have a dichotomous dependent variable and a set of explanatory variables). Unlike in probit regression, GEE probit allows for dependence within clusters, such as in longitudinal data, although its use is not

limited to just panel data. The user must first specify a “working” correlation matrix for the clusters, which models the dependence of each observation with other observations in the same cluster. The “working” correlation matrix is a  $T \times T$  matrix of correlations, where  $T$  is the size of the largest cluster and the elements of the matrix are correlations between within-cluster observations. The appeal of GEE models is that it gives consistent estimates of the parameters and consistent estimates of the standard errors can be obtained using a robust “sandwich” estimator even if the “working” correlation matrix is incorrectly specified. If the “working” correlation matrix is correctly specified, GEE models will give more efficient estimates of the parameters. GEE models measure population-averaged effects as opposed to cluster-specific effects (See (author?) [4]).

### 5.0.25 Syntax

```
> z.out <- zelig(Y ~ X1 + X2, model = "probit.gee",
  id = "X3", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

where `id` is a variable which identifies the clusters. The data should be sorted by `id` and should be ordered within each cluster when appropriate.

### 5.0.26 Additional Inputs

- **robust**: defaults to `TRUE`. If `TRUE`, consistent standard errors are estimated using a “sandwich” estimator.

Use the following arguments to specify the structure of the “working” correlations within clusters:

- **corstr**: defaults to `"independence"`. It can take on the following arguments:
  - Independence (`corstr = "independence"`):  $\text{cor}(y_{it}, y_{it'}) = 0, \forall t, t'$  with  $t \neq t'$ . It assumes that there is no correlation within the clusters and the model becomes equivalent to standard probit regression. The “working” correlation matrix is the identity matrix.
  - Fixed (`corstr = "fixed"`): If selected, the user must define the “working” correlation matrix with the `R` argument rather than estimating it from the model.
  - Stationary  $m$  dependent (`corstr = "stat_M_dep"`):

$$\text{cor}(y_{it}, y_{it'}) = \begin{cases} \alpha_{|t-t'|} & \text{if } |t - t'| \leq m \\ 0 & \text{if } |t - t'| > m \end{cases}$$

If (`corstr = "stat_M_dep"`), you must also specify `Mv = m`, where  $m$  is the number of periods  $t$  of dependence. Choose this option when

the correlations are assumed to be the same for observations of the same  $|t - t'|$  periods apart for  $|t - t'| \leq m$ .

Sample “working” correlation for Stationary 2 dependence ( $Mv=2$ )

$$\begin{pmatrix} 1 & \alpha_1 & \alpha_2 & 0 & 0 \\ \alpha_1 & 1 & \alpha_1 & \alpha_2 & 0 \\ \alpha_2 & \alpha_1 & 1 & \alpha_1 & \alpha_2 \\ 0 & \alpha_2 & \alpha_1 & 1 & \alpha_1 \\ 0 & 0 & \alpha_2 & \alpha_1 & 1 \end{pmatrix}$$

– Non-stationary  $m$  dependent (`corstr` = "non\_stat\_M\_dep"):

$$\text{cor}(y_{it}, y_{it'}) = \begin{cases} \alpha_{tt'} & \text{if } |t - t'| \leq m \\ 0 & \text{if } |t - t'| > m \end{cases}$$

If (`corstr` = "non\_stat\_M\_dep"), you must also specify  $Mv = m$ , where  $m$  is the number of periods  $t$  of dependence. This option relaxes the assumption that the correlations are the same for all observations of the same  $|t - t'|$  periods apart.

Sample “working” correlation for Non-stationary 2 dependence ( $Mv=2$ )

$$\begin{pmatrix} 1 & \alpha_{12} & \alpha_{13} & 0 & 0 \\ \alpha_{12} & 1 & \alpha_{23} & \alpha_{24} & 0 \\ \alpha_{13} & \alpha_{23} & 1 & \alpha_{34} & \alpha_{35} \\ 0 & \alpha_{24} & \alpha_{34} & 1 & \alpha_{45} \\ 0 & 0 & \alpha_{35} & \alpha_{45} & 1 \end{pmatrix}$$

– Exchangeable (`corstr` = "exchangeable"):  $\text{cor}(y_{it}, y_{it'}) = \alpha, \forall t, t'$  with  $t \neq t'$ . Choose this option if the correlations are assumed to be the same for all observations within the cluster.

Sample “working” correlation for Exchangeable

$$\begin{pmatrix} 1 & \alpha & \alpha & \alpha & \alpha \\ \alpha & 1 & \alpha & \alpha & \alpha \\ \alpha & \alpha & 1 & \alpha & \alpha \\ \alpha & \alpha & \alpha & 1 & \alpha \\ \alpha & \alpha & \alpha & \alpha & 1 \end{pmatrix}$$

– Stationary  $m$ th order autoregressive (`corstr` = "AR-M"): If (`corstr` = "AR-M"), you must also specify  $Mv = m$ , where  $m$  is the number of periods  $t$  of dependence. For example, the first order autoregressive model (AR-1) implies  $\text{cor}(y_{it}, y_{it'}) = \alpha^{|t-t'|}, \forall t, t'$  with  $t \neq t'$ . In AR-1, observation 1 and observation 2 have a correlation of  $\alpha$ . Observation 2 and observation 3 also have a correlation of  $\alpha$ . Observation 1 and observation 3 have a correlation of  $\alpha^2$ , which is a



function of how 1 and 2 are correlated ( $\alpha$ ) multiplied by how 2 and 3 are correlated ( $\alpha$ ). Observation 1 and 4 have a correlation that is a function of the correlation between 1 and 2, 2 and 3, and 3 and 4, and so forth.

Sample “working” correlation for Stationary AR-1 ( $Mv=1$ )

$$\begin{pmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ \alpha & 1 & \alpha & \alpha^2 & \alpha^3 \\ \alpha^2 & \alpha & 1 & \alpha & \alpha^2 \\ \alpha^3 & \alpha^2 & \alpha & 1 & \alpha \\ \alpha^4 & \alpha^3 & \alpha^2 & \alpha & 1 \end{pmatrix}$$

– Unstructured (`corstr = "unstructured"`):  $\text{cor}(y_{it}, y_{it'}) = \alpha_{tt'}, \forall t, t'$  with  $t \neq t'$ . No constraints are placed on the correlations, which are then estimated from the data.

- `Mv`: defaults to 1. It specifies the number of periods of correlation and only needs to be specified when `corstr` is `"stat_M_dep"`, `"non_stat_M_dep"`, or `"AR-M"`.
- `R`: defaults to `NULL`. It specifies a user-defined correlation matrix rather than estimating it from the data. The argument is used only when `corstr` is `"fixed"`. The input is a  $T \times T$  matrix of correlations, where  $T$  is the size of the largest cluster.

### 5.0.27 Examples

1. Example with Stationary 3 Dependence

Attaching the sample turnout dataset:

```
> data(turnout)
```

Variable identifying clusters

```
> turnout$cluster <- rep(c(1:200),10)
```

Sorting by cluster

```
> sorted.turnout <- turnout[order(turnout$cluster),]
```

Estimating parameter values:

```
> z.out1 <- zelig(vote ~ race + educate, model = "probit.gee", id = "cluster", data = sorted.turnout)
```

```
(Intercept)  racewhite    educate
-0.72594913  0.29907642  0.09711897
```

How to cite this model in Zelig:

Patrick Lam. 2013.  
 "probit.gee: General Estimating Equation for Poisson Regression"  
 in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software,"  
<http://gking.harvard.edu/zelig>

Setting values for the explanatory variables to their default values:

```
> x.out1 <- setx(z.out1)
```

Simulating quantities of interest:

```
> s.out1 <- sim(z.out1, x = x.out1)
```

```
> summary(s.out1)
```

Model: probit.gee

Number of simulations: 1000

Values of X

(Intercept)	racewhite	educate
1	1	1 12.06675

```
attr("assign")
```

```
[1] 0 1 2
```

```
attr("contrasts")
```

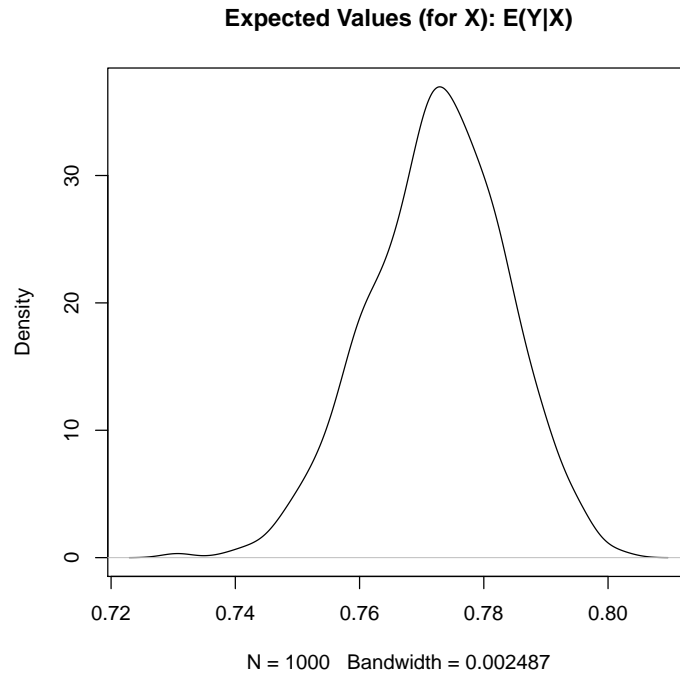
```
attr("contrasts")$race
```

```
[1] "contr.treatment"
```

Expected Values (for x):  $E(Y|X)$

mean	sd	50%	2.5%	97.5%
0.773	0.011	0.773	0.75	0.793

```
> plot(s.out1)
```



## 2. Simulating First Differences

Estimating the risk difference (and risk ratio) between low education (25th percentile) and high education (75th percentile) while all the other variables held at their default values.

```
> x.high <- setx(z.out1, educate = quantile(turnout$educate, prob = 0.75))
> x.low <- setx(z.out1, educate = quantile(turnout$educate, prob = 0.25))

> s.out2 <- sim(z.out1, x = x.high, x1 = x.low)
```

```
> summary(s.out2)
```

Model: probit.gee

Number of simulations: 1000

Values of X

(Intercept)	racewhite	educate
1	1	14

```
attr("assign")
```

```
[1] 0 1 2
```

```
attr("contrasts")
```

```
attr("contrasts")$race
```

```
[1] "contr.treatment"
```

```
Values of X1
  (Intercept) racewhite educate
1           1           1      10
attr("assign")
[1] 0 1 2
attr("contrasts")
attr("contrasts")$race
[1] "contr.treatment"
```

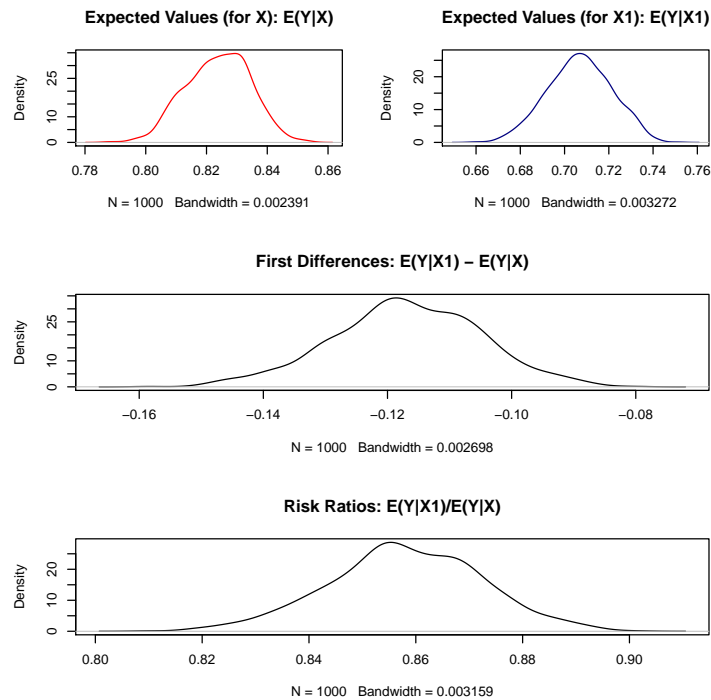
```
Expected Values (for x): E(Y|X)
  mean    sd  50%  2.5% 97.5%
0.824 0.011 0.824 0.804 0.843
```

```
Expected Values (for x1): E(Y|X1)
  mean    sd  50%  2.5% 97.5%
0.707 0.014 0.707 0.678 0.733
```

```
First Differences: E(Y|X1) - E(Y|X)
  mean    sd  50%  2.5% 97.5%
-0.117 0.012 -0.117 -0.141 -0.093
```

```
Risk Ratios: E(Y|X1)/E(Y|X)
  mean    sd  50%  2.5% 97.5%
0.858 0.014 0.858 0.829 0.886
```

```
> plot(s.out2)
```



### 3. Example with Fixed Correlation Structure

User-defined correlation structure

```
> corr.mat <- matrix(rep(0.5,100), nrow=10, ncol=10)
> diag(corr.mat) <- 1
```

Generating empirical estimates:

```
> z.out2 <- zelig(vote ~ race + educate, model = "probit.gee", id = "cluster", data = s

(Intercept)    racewhite    educate
-0.72594913  0.29907642  0.09711897
```

How to cite this model in Zelig:

Patrick Lam. 2013.

"probit.gee: General Estimating Equation for Poisson Regression"

in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software,"  
<http://gking.harvard.edu/zelig>

Viewing the regression output:

```
> summary(z.out2)
```

GEE: GENERALIZED LINEAR MODELS FOR DEPENDENT DATA  
 gee S-function, version 4.13 modified 98/01/27 (1998)

Model:  
 Link: Probit  
 Variance to Mean Relation: Binomial  
 Correlation Structure: Fixed

Call:  
 gee(formula = formula, id = id, corstr = corstr, family = binomial(link = "probit"),  
 data = data, R = R, .hook = robust.gee.hook)

Summary of Residuals:

	Min	1Q	Median	3Q	Max
	-0.9191419	-0.3146504	0.2063033	0.2349483	0.7801544

Coefficients:

	Estimate	Naive S.E.	Naive z	Robust S.E.	Robust z
(Intercept)	-0.77271488	0.105618565	-7.316090	0.133841982	-5.773337
racewhite	0.33534707	0.061921463	5.415684	0.088349410	3.795691
educate	0.09666793	0.007082234	13.649355	0.009711359	9.954110

Estimated Scale Parameter: 0.9734069  
 Number of Iterations: 3

Working Correlation

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
[1,]	1.0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
[2,]	0.5	1.0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
[3,]	0.5	0.5	1.0	0.5	0.5	0.5	0.5	0.5	0.5	0.5
[4,]	0.5	0.5	0.5	1.0	0.5	0.5	0.5	0.5	0.5	0.5
[5,]	0.5	0.5	0.5	0.5	1.0	0.5	0.5	0.5	0.5	0.5
[6,]	0.5	0.5	0.5	0.5	0.5	1.0	0.5	0.5	0.5	0.5
[7,]	0.5	0.5	0.5	0.5	0.5	0.5	1.0	0.5	0.5	0.5
[8,]	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1.0	0.5	0.5
[9,]	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1.0	0.5
[10,]	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1.0

### 5.0.28 The Model

Suppose we have a panel dataset, with  $Y_{it}$  denoting the binary dependent variable for unit  $i$  at time  $t$ .  $Y_i$  is a vector or cluster of correlated data where  $y_{it}$  is correlated with  $y_{it'}$  for some or all  $t, t'$ . Note that the model assumes correlations within  $i$  but independence across  $i$ .

- The *stochastic component* is given by the joint and marginal distributions

$$\begin{aligned} Y_i &\sim f(y_i \mid \pi_i) \\ Y_{it} &\sim g(y_{it} \mid \pi_{it}) \end{aligned}$$

where  $f$  and  $g$  are unspecified distributions with means  $\pi_i$  and  $\pi_{it}$ . GEE models make no distributional assumptions and only require three specifications: a mean function, a variance function, and a correlation structure.

- The *systematic component* is the *mean function*, given by:

$$\pi_{it} = \Phi(x_{it}\beta)$$

where  $\Phi(\mu)$  is the cumulative distribution function of the Normal distribution with mean 0 and unit variance,  $x_{it}$  is the vector of  $k$  explanatory variables for unit  $i$  at time  $t$  and  $\beta$  is the vector of coefficients.

- The *variance function* is given by:

$$V_{it} = \pi_{it}(1 - \pi_{it})$$

- The *correlation structure* is defined by a  $T \times T$  “working” correlation matrix, where  $T$  is the size of the largest cluster. Users must specify the structure of the “working” correlation matrix *a priori*. The “working” correlation matrix then enters the variance term for each  $i$ , given by:

$$V_i = \phi A_i^{\frac{1}{2}} R_i(\alpha) A_i^{\frac{1}{2}}$$

where  $A_i$  is a  $T \times T$  diagonal matrix with the variance function  $V_{it} = \pi_{it}(1 - \pi_{it})$  as the  $t$ th diagonal element,  $R_i(\alpha)$  is the “working” correlation matrix, and  $\phi$  is a scale parameter. The parameters are then estimated via a quasi-likelihood approach.

- In GEE models, if the mean is correctly specified, but the variance and correlation structure are incorrectly specified, then GEE models provide consistent estimates of the parameters and thus the mean function as well, while consistent estimates of the standard errors can be obtained via a robust “sandwich” estimator. Similarly, if the mean and variance are correctly specified but the correlation structure is incorrectly specified, the parameters can be estimated consistently and the standard errors can be estimated consistently with the sandwich estimator. If all three are specified correctly, then the estimates of the parameters are more efficient.
- The robust “sandwich” estimator gives consistent estimates of the standard errors when the correlations are specified incorrectly only if the number of units  $i$  is relatively large and the number of repeated periods  $t$  is relatively small. Otherwise, one should use the “naïve” model-based standard errors, which assume that the specified correlations are close approximations to the true underlying correlations. See [?] for more details.

### 5.0.29 Quantities of Interest

- All quantities of interest are for marginal means rather than joint means.
- The method of bootstrapping generally should not be used in GEE models. If you must bootstrap, bootstrapping should be done within clusters, which is not currently supported in Zelig. For conditional prediction models, data should be matched within clusters.
- The expected values (`qi$ev`) for the GEE probit model are simulations of the predicted probability of a success:

$$E(Y) = \pi_c = \Phi(x_c\beta),$$

given draws of  $\beta$  from its sampling distribution, where  $x_c$  is a vector of values, one for each independent variable, chosen by the user.

- The first difference (`qi$fd`) for the GEE probit model is defined as

$$FD = \Pr(Y = 1 \mid x_1) - \Pr(Y = 1 \mid x).$$

- The risk ratio (`qi$rr`) is defined as

$$RR = \Pr(Y = 1 \mid x_1) / \Pr(Y = 1 \mid x).$$

- In conditional prediction models, the average expected treatment effect (`att.ev`) for the treatment group is

$$\frac{1}{\sum_{i=1}^n \sum_{t=1}^T tr_{it}} \sum_{i:tr_{it}=1}^n \sum_{t:tr_{it}=1}^T \{Y_{it}(tr_{it} = 1) - E[Y_{it}(tr_{it} = 0)]\},$$

where  $tr_{it}$  is a binary explanatory variable defining the treatment ( $tr_{it} = 1$ ) and control ( $tr_{it} = 0$ ) groups. Variation in the simulations are due to uncertainty in simulating  $E[Y_{it}(tr_{it} = 0)]$ , the counterfactual expected value of  $Y_{it}$  for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to  $tr_{it} = 0$ .

### 5.0.30 Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run `z.out <- zelig(y ~ x, model = "probit.gee", id, data)`, then you may examine the available information in `z.out` by using `names(z.out)`, see the `coefficients` by using `z.out$coefficients`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output object `z.out`, you may extract:



- `coefficients`: parameter estimates for the explanatory variables.
  - `residuals`: the working residuals in the final iteration of the fit.
  - `fitted.values`: the vector of fitted values for the systemic component,  $\pi_{it}$ .
  - `linear.predictors`: the vector of  $x_{it}\beta$
  - `max.id`: the size of the largest cluster.
- From `summary(z.out)`, you may extract:
    - `coefficients`: the parameter estimates with their associated standard errors,  $p$ -values, and  $z$ -statistics.
    - `working.correlation`: the “working” correlation matrix
  - From the `sim()` output object `s.out`, you may extract quantities of interest arranged as matrices indexed by simulation  $\times$  `x`-observation (for more than one `x`-observation). Available quantities are:
    - `qi$ev`: the simulated expected probabilities for the specified values of `x`.
    - `qi$fd`: the simulated first difference in the expected probabilities for the values specified in `x` and `x1`.
    - `qi$rr`: the simulated risk ratio for the expected probabilities simulated from `x` and `x1`.
    - `qi$att.ev`: the simulated average expected treatment effect for the treated from conditional prediction models.

## How To Cite *probit.gee* Zelig model

Patrick Lam. 2007. “probit.gee: Generalized Estimating Equation for Probit Regression,” in Kosuke Imai, Gary King, and Olivia Lau, “Zelig: Everyone’s Statistical Software,” <http://gking.harvard.edu/zelig>.

## How to Cite the Zelig Software Package

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Imai, Kosuke, Gary King, and Olivia Lau. (2008). “Toward A Common Framework for Statistical Analysis and Development.” *Journal of Computational and Graphical Statistics*, Vol. 17, No. 4 (December), pp. 892-913.

## See also

The `gee` function is part of the `gee` package by Vincent J. Carey, ported to R by Thomas Lumley and Brian Ripley. Advanced users may wish to refer to `help(gee)` and `help(family)`. Sample data are from [2].

## References

- [1] Gary King, James Alt, Nancy Burns, and Michael Laver. A unified model of cabinet dissolution in parliamentary democracies. *American Journal of Political Science*, 34(3):846–871, August 1990. <http://gking.harvard.edu/files/abs/coal-abs.shtml>.
- [2] Gary King, Michael Tomz, and Jason Wittenberg. Making the most of statistical analyses: Improving interpretation and presentation. *American Journal of Political Science*, 44(2):341–355, April 2000. <http://gking.harvard.edu/files/abs/making-abs.shtml>.
- [3] Lisa Martin. *Coercive Cooperation: Explaining Multilateral Economic Sanctions*. Princeton University Press, 1992. Please inquire with Lisa Martin before publishing results from these data, as this dataset includes errors that have since been corrected.
- [4] Christopher Zorn. Generalized estimating equation models for correlated data: A review with applications. *American Journal of Political Science*, 45:470–490, April 2001.