# Introduction to cmaRs

This package is designed to construct Conic Multivariate Adaptive Regression Splines (CMARS) in R. CMARS model is a linear combination of basis functions that is taken from the forward part of the MARS algorithm. CMARS model parameters are obtained after solving Conic Quadratic Programming (CQP) which includes two cone constraints to handle accuracy and complexity of the model. Please check Weber, et al. (2011) CMARS: a new contribution to nonparametric regression with multivariate adaptive regression splines supported by continuous optimization. Inverse Problems in Science and Engineering, 2012, 20.3: 371-400. doi:10.1080/17415977.2011.62477, for more details.

Prediction and binary classification models can be constructed with cmaRs. Note that **in order to construct CMARS models, both MOSEK software and Rmosek package needed. Please follow carefully the steps available in https://docs.mosek.com/latest/rmosek/install-interface.html for successful installation.**

In order to construct CMARS models, a user can define the following arguments

- **formula**: description of the model
- **degree**: maximum degree of interaction
- **nk**: maximum number of model terms before pruning
- **data**: data frame
- **classification**: binary variable indicating whether the model is for prediction or binary classification
- **threshold.class**: the threshold value that is used to convert probabilities to classes in binary classification models.

```
library(cmaRs)
```

This package includes three data sets; preddata.std, classdata.std and table.b6. The first two data sets are taken from UCI: Machine Learning Repository (available at http://archive.ics.uci.edu/ml/), preprocessed and standardized. The first one is used for prediction and the other one is used for binary classification. Moreover, there is one more data set called as table.b6 which is directly taken from the "MPV" package (version 1.58) (Braun, W. J., and MacQueen, S., 2022).

- The preddata.std includes 103 observations and 8 variables.

```
data(preddata.std)
head(preddata.std)
#>       Cement       Slag     Fly_ash      Water        SP Coarse_Aggr
#> 1  0.5464926 0.06659151 -0.51528392  0.6349930 0.1639143   0.2265080
#> 2 -0.8480797 1.17473721  0.49152869 -0.8495560 1.2324693  -0.4636044
#> 3 -0.8607576 1.15819773  0.49152869 -0.8990409 2.6572093  -0.4975444
#> 4 -0.8607576 1.15819773  0.47982157 -0.8990409 3.7257643  -0.5201710
#> 5 -0.9621810 0.56277615 -0.05870611  1.1298427 0.5200993   0.4414611
#> 6 -1.0509265 0.18236793 -0.39821269  0.2391133 0.1639143  -0.2712780
#>     Fine_Aggr Compressive_Strength
#> 1 -0.94099877           -0.1338845
#> 2  0.10096198            0.6507313
```

```
#> 3  0.05360013           0.7362097
#> 4  0.02202556           0.7706563
#> 5 -1.28831902          -1.1762114
#> 6  1.41130656          -1.3816148
```

- The classdata.std includes 569 observations and 31 variables.

```
data(classdata.std)
head(classdata.std)
#>   y         x1         x2         x3         x4         x5         x6
#> 1 1  1.0960995 -2.0715123  1.2688173  0.9835095  1.5670875  3.2806281
#> 2 1  1.8282120 -0.3533215  1.6844726  1.9070303 -0.8262354 -0.4866435
#> 3 1  1.5784992  0.4557859  1.5651260  1.5575132  0.9413821  1.0519999
#> 4 1 -0.7682333  0.2535091 -0.5921661 -0.7637917  3.2806668  3.3999174
#> 5 1  1.7487579 -1.1508038  1.7750113  1.8246238  0.2801253  0.5388663
#> 6 1 -0.4759559 -0.8346009 -0.3868077 -0.5052059  2.2354545  1.2432416
#>            x7        x8           x9        x10        x11        x12
#> 1  2.65054179 2.5302489  2.215565542  2.2537638  2.4875451 -0.5647681
#> 2 -0.02382489 0.5476623  0.001391139 -0.8678888  0.4988157 -0.8754733
#> 3  1.36227979 2.0354398  0.938858720 -0.3976580  1.2275958 -0.7793976
#> 4  1.91421287 1.4504311  2.864862154  4.9066020  0.3260865 -0.1103120
#> 5  1.36980615 1.4272370 -0.009552062 -0.5619555  1.2694258 -0.7895490
#> 6  0.86554001 0.8239307  1.004517928  1.8883435 -0.2548461 -0.5921406
#>          x13        x14        x15         x16        x17         x18
#> 1  2.8305403  2.4853907 -0.2138135  1.31570389  0.7233897  0.66023900
#> 2  0.2630955  0.7417493 -0.6048187 -0.69231710 -0.4403926  0.25993335
#> 3  0.8501802  1.1802975 -0.2967439  0.81425704  0.2128891  1.42357487
#> 4  0.2863415 -0.2881246  0.6890953  2.74186785  0.8187979  1.11402678
#> 5  1.2720701  1.1893103  1.4817634 -0.04847723  0.8277425  1.14319885
#> 6 -0.3210217 -0.2890039  0.1562093  0.44515196  0.1598845 -0.06906279
#>          x19         x20        x21         x22        x23        x24
#> 1  1.1477468  0.90628565  1.8850310 -1.35809849  2.3015755  1.9994782
#> 2 -0.8047423 -0.09935632  1.8043398 -0.36887865  1.5337764  1.8888270
#> 3  0.2368272  0.29330133  1.5105411 -0.02395331  1.3462906  1.4550043
#> 4  4.7285198  2.04571087 -0.2812170  0.13386631 -0.2497196 -0.5495377
#> 5 -0.3607748  0.49888916  1.2974336 -1.46548091  1.3373627  1.2196511
#> 6  0.1340009  0.48641784 -0.1653528 -0.31356043 -0.1149083 -0.2441054
#>          x25        x26        x27       x28        x29        x30
#> 1  1.3065367  2.6143647  2.1076718 2.2940576  2.7482041  1.9353117
#> 2 -0.3752817 -0.4300658 -0.1466200 1.0861286 -0.2436753  0.2809428
#> 3  0.5269438  1.0819801  0.8542223 1.9532817  1.1512420  0.2012142
#> 4  3.3912907  3.8899747  1.9878392 2.1738732  6.0407261  4.9306719
#> 5  0.2203623 -0.3131190  0.6126397 0.7286181 -0.8675896 -0.3967505
#> 6  2.0467119  1.7201029  1.2621327 0.9050914  1.7525273  2.2398308
```

- The table.b6 includes 28 observations and 5 variables.

```
data(table.b6)
head(table.b6)
#>          y     x1    x2     x3     x4
#> 1 0.000450 0.0105  90.9 0.0164 0.0177
#> 2 0.000450 0.0110  84.6 0.0165 0.0172
```

```
#> 3 0.000473 0.0106  88.9 0.0164 0.0157
#> 4 0.000507 0.0116 488.7 0.0187 0.0082
#> 5 0.000457 0.0121 454.4 0.0187 0.0070
#> 6 0.000452 0.0123 439.2 0.0187 0.0065
```

## The Prediction Modeling

As an example, the following prediction CMARS model for the data set trees can be constructed as follows.

```
prediction.model <-  cmaRs(Volume ~ ., degree = 2, nk = 20, data = trees)
```

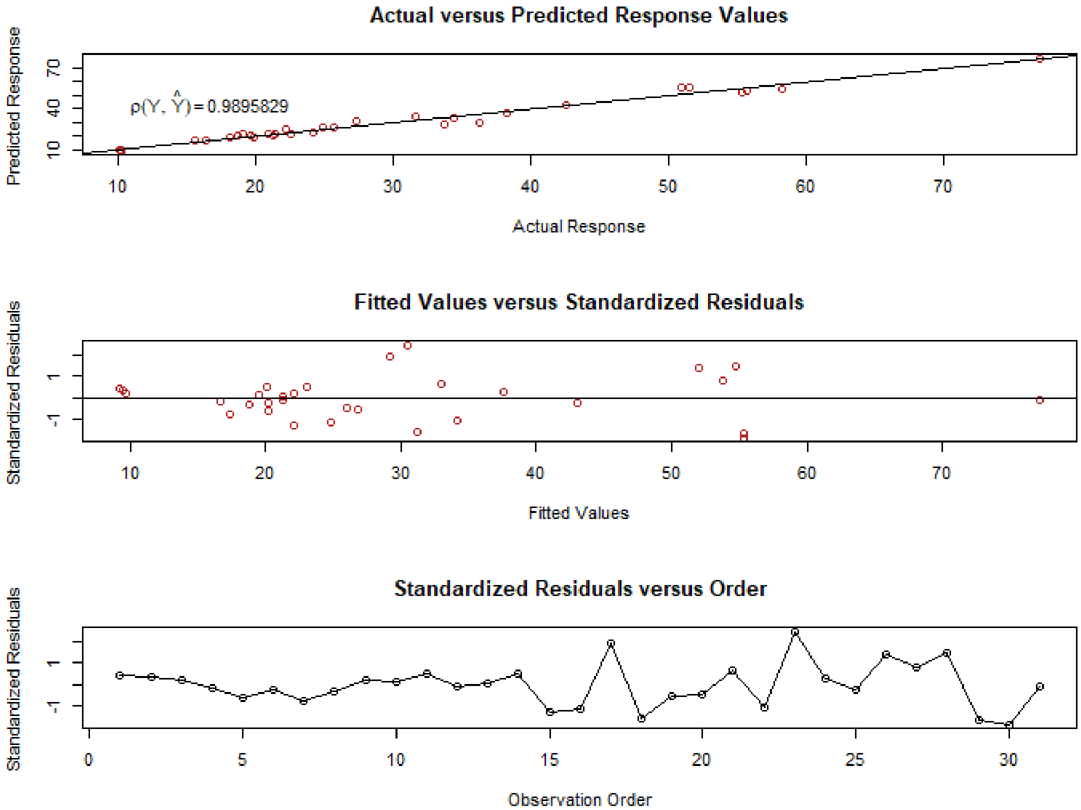
In order to study this model in detail, the **summary** function can be used. Note here that the CMARS model is determined with **nk** = 20 and **degree** = 2. Here, the final model contains six basis function in total with one interaction term.

```
summary(prediction.model)
#> Call:
#> cmaRs(formula = Volume ~ ., data = trees, degree = 2, nk = 20)

#> Volume = +29.1634
#> +4.9278 * pmax(0,Girth-14.2)
#> -3.2309 * pmax(0,14.2-Girth)
#> +0.7313 * pmax(0,Height-75)
#> -0.1684 * pmax(0,75-Height)
#> +0.1312 * pmax(0,Girth-8.3)*pmax(0,Height-75)
#> -1.2977 * pmax(0,Height-78)

#> R2 0.9793 r 0.9896 RSS 168.0029
```

Some performance measures are also printed at the end of the output. For instance, the $R^2$, r and RSS values are given for the prediction models.

It is also possible to construct several graphs of a prediction model. Here, **plot** function helps for this.

```
fig1 <- plot(prediction.model)
```




Another function of this package is **predict** which calculates the fitted values of a CMARS model. An example is given below.

```
predict(prediction.model)
#> [1] 9.259021 9.386200 9.695543 16.703820 20.239753 20.164926 17.308760
#> [8] 18.824519 22.046053 19.470698 22.996184 21.255036 21.255036 20.075638
#> [15] 22.055412 24.794797 29.229259 31.136370 26.874258 26.018423 32.955379
#> [22] 34.096100 30.473296 37.528153 43.074254 52.021298 53.805352 54.756953
#> [29] 55.315354 55.315354 77.168803
```

# The Classification Modeling

cmaRs can also construct binary classification models. An example is given below.

```
library(earth)
library(cmaRs)
classification.model <- cmaRs(survived ~ age, nk = 35,
                        classification = TRUE, data = etitanic)
```

Note that, the **classification** argument is set as TRUE here which indicates a binary classification model. Moreover, the degree argument is used as 1 which is its default value indicating a main effect model. This model is constructed by using the only continuous variable in the data set, age. Similar to the previous example, the **summary** function presents the details of the model.

```
summary(classification.model)
#> Call:
#> cmaRs(formula = survived ~ age, data = etitanic, classification = TRUE,
#> nk = 35)
#>
#> survived = -4.9489
#> -0.3084 * pmax(0,age-18)
#> +0.3321 * pmax(0,18-age)
#> -0.427 * pmax(0,age-53)
#> +0.3564 * pmax(0,age-67)
#> -0.3291 * pmax(0,age-64)
#> +1.1664 * pmax(0,age-46)
#> +0.7742 * pmax(0,age-57)
#> -0.1451 * pmax(0,age-35)
#> -0.7469 * pmax(0,age-58)
#> +0.2288 * pmax(0,age-61)
#> +0.0725 * pmax(0,age-41)
#> -1.0824 * pmax(0,age-45)
#> -0.5147 * pmax(0,age-48)
#> +0.3658 * pmax(0,age-51)
#> +0.281 * pmax(0,age-44)
#> +0.0994 * pmax(0,age-34)
#> +0.6316 * pmax(0,age-2)
#> -0.32 * pmax(0,age-3)
#>
#> AUC 0.6221 MCR 0.3681 PCC 0.6319 precision 0.6339 recall 0.895 specificity 0.2506
```

The model, here, includes terms with no interactions. Moreover, some performance measures for this binary classification are given in the last line. Some of them such as "Misclassification Rate" is calculated by taking the threshold as 0.5 as the default value.

Another value of the threshold can also be set by using the argument, **threshold.class**, available in the cmaRs function. The following example is given to examplify this.

```
classification.model.threshold <- cmaRs(survived ~ age, nk = 35,
                                classification = TRUE, data = etitanic,
                                threshold.class = 0.1, Auto.linpreds = FALSE)
```

The second model follows the same construction steps but with different performance values.

```
summary(classification.model.threshold)
#> Call:
#> cmaRs(formula = survived ~ age, data = etitanic, classification = TRUE,
#> threshold.class = 0.1, nk = 35, Auto.linpreds = FALSE)
#>
#> survived = -4.9489
#> -0.3084 * pmax(0,age-18)
#> +0.3321 * pmax(0,18-age)
#> -0.427 * pmax(0,age-53)
#> +0.3564 * pmax(0,age-67)
#> -0.3291 * pmax(0,age-64)
#> +1.1664 * pmax(0,age-46)
#> +0.7742 * pmax(0,age-57)
```

```
#> -0.1451 * pmax(0,age-35)
#> -0.7469 * pmax(0,age-58)
#> +0.2288 * pmax(0,age-61)
#> +0.0725 * pmax(0,age-41)
#> -1.0824 * pmax(0,age-45)
#> -0.5147 * pmax(0,age-48)
#> +0.3658 * pmax(0,age-51)
#> +0.281 * pmax(0,age-44)
#> +0.0994 * pmax(0,age-34)
#> +0.6316 * pmax(0,age-2)
#> -0.32 * pmax(0,age-3)
#>
#> AUC 0.6221 MCR 0.5746 PCC 0.4254 precision 1 recall 0.0291 specificity 1
```

In addition to the performance values of the models, it is also possible to construct the ROC curve for the classification model using the **plot** function as follows.

```
fig2 <- plot(classification.model)
```